6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11 – 15 June 2018, Glasgow, UK

Geometrically Exact Finite Element Formulations for Highly Slender Beams and Their Interaction: Kirchhoff-Love Theory vs. Simo-Reissner Theory

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Key Words: *Geometrically Exact Beams; Kirchhoff-Love Theory; Simo-Reissner Theory;*

Highly slender fiber- or rod-like components represent essential constituents of mechanical systems in countless fields of application. Examples are high-tensile industrial ropes and webbings, fiber-reinforced composite materials or synthetic polymer materials. On entirely different time and length scales, such slender components are relevant when analyzing the supercoiling process of DNA strands, the characteristics of carbon nanotubes or the Brownian dynamics within the cytoskeleton of biological cells. In this work, finite element formulations for the modeling of such highly slender mechanical members and their contact interaction are proposed on the basis of the geometrically exact Kirchhoff-Love beam theory.

Compared to other classes of geometrically nonlinear beam elements, geometrically exact beam element formulations are characterized by a highest degree of accuracy and computational efficiency [1]. While the existing representatives are almost exclusively based on the Simo-Reissner theory of shear-deformable beams, the current work proposes novel finite elements based on the geometrically exact Kirchhoff-Love theory of thin beams. The proposed formulations are the first of this category that account for curved 3D initial geometries with anisotropic cross-section shapes and fulfill the fundamental mechanical properties of objectivity and path independence. For finite elements derived from 3D Boltzmann continua such properties are standard. However, the non-additivity and non-commutativity of the configuration space underlying geometrically exact beams, comprising the nonlinear manifold SO(3), requires special interpolation strategies [2]. Thereto, novel orthonormal rotation interpolation schemes are proposed that eventually yield two alternative beam elements based on a strong and a weak enforcement of the Kirchhoff constraint, respectively [3]. Compared to existing formulations of Simo-Reissner type, it is shown that these Kirchhoff-Love beam elements allow for numerical advantages in the range of high beam slenderness ratios such as lower discretization error level per degree of freedom or improved performance of nonlinear solvers. Eventually, the developed beam elements are combined with a recently proposed beam-to-beam contact scheme in order to analyze complex mechanical systems consisting of highly slender fibers with arbitrary orientation [4].

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