An optimization-based discontinuous Galerkin approach for high-order accurate shock tracking

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We introduce a high-order accurate, nonlinearly stable numerical framework for solving steady conservation laws with discontinuous solution features such as shock waves [1, 2]. The method falls into the category of a shock tracking or *r*-adaptive method and is based on the observation that numerical discretizations such as finite volume or discontinuous Galerkin methods that support discontinuities along element faces can perfectly represent discontinuities and provide appropriate stabilization through approximate Riemann solvers. The difficulty lies in aligning element faces with the unknown discontinuity. The proposed method recasts a discretized conservation law as a PDE-constrained optimization problem whose solution is a (curved) mesh that tracks the discontinuity and the solution of the discrete conservation law on this mesh. The discrete state vector and nodal positions of the high-order mesh are taken as optimization variables. The objective function is a discontinuity indicator that monotonically approaches a minimum as element faces approach the shock surface. The discretized conservation law on a parametrized domain defines the equality constraints for the optimization problem. A full space optimization solver is used to simultaneously converge the state vector and mesh to their optimal values. This ensures the solution of the discrete PDE is never required on meshes that are not aligned with discontinuities and improves nonlinear stability. The method is demonstrated on a number of one- and two-dimensional transonic and supersonic flow problems. In all cases, the framework tracks the discontinuity closely with curved mesh elements and provides accurate solutions on extremely coarse meshes.

REFERENCES

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