6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11 15 June 2018, Glasgow, UK

SEMI-ANALYTICAL METHOD FOR VIBRATION OF GENERAL ROTATING SHELLS OF REVOLUTION

Kedar S. Vaidya[1] **and Robert G. Parker**[2]

[1] Virginia Tech, Blacksburg, VA 24061, USA, kedarv93@vt.edu
[2] Virginia Tech, Blacksburg, VA 24061, USA, r.parker@vt.edu

Key words: *Fourier series, rotating, vibration, finite element method, general shell theory*

Rotating structures are encountered in many mechanical systems, such as turbomachinery and mechanical transmission systems. To make lightweight structures, thin-walled shell structures are widely used. Thus, the study of rotating shell structures is important. A general thin shell theory for shells of revolution can model conical and cylindrical shells, circular plates, rings, etc. Shells are the building blocks for complex thin-walled structures.

Spinning systems coupled to space-fixed structures (for example, the contacting gear teeth in thin-walled gears) have contact points that shift in time, making them difficult to model in a rotating reference frame by conventional finite element methods. This problem can be alleviated for axisymmetric rotating structures by performing the analysis in the stationary reference frame. The finite element discretization of the axisymmetric structure is carried out on the cross section in the r-z plane. The displacement fields are approximated as Fourier series in 'θ' [1]. The displacement field on an element with 'H' harmonics is $q(s,\theta,t) = \boldsymbol{\psi_q(s) q_0(t)} + \sum_{i=1}^{H} [\boldsymbol{\psi_q(s) q_{ic}(t)} \cos(i\theta) + \boldsymbol{\psi_q(s) q_{is}(t)} \sin(i\theta)]$. Linear interpolation functions are used for $\boldsymbol{\psi(s)}$ along the directions in the tangent plane of the shell. In the normal direction, continuity of both displacement and slope are required, thus, cubic Hermite interpolation functions are used.

In this study, Sanders nonlinear general shell theory [2] is adopted and applied to a broad class of general shells of revolution. Sanders strain-displacement relations with constitutive equations are used to derive the strain energy. The kinetic energy of the rotating shell is calculated from the material time derivative of the position vector. Euler-Lagrange mechanics yields the equations of motion in terms of matrices with the nodal coefficients in the Fourier series as unknowns. The centrifugal forcing induced by rotation leads to steady deflection. This steady deflection introduces an additional tension in the stiffness matrix obtained from the nonlinear terms in the strain-displacement equations. The matrix formulation has a standard conservative, gyroscopic structure form with symmetric and skew-symmetric matrices. Coriolis, centripetal, and steady tension effects are included.

The eigenvalue problem, $\mathbf{M}\ddot{\boldsymbol{q}} + 2\Omega\mathbf{G}\dot{\boldsymbol{q}} + (\mathbf{K} + \mathbf{K_{st}} - \Omega^2\mathbf{C})\boldsymbol{q} = \mathbf{0}$, is solved to find natural frequencies and vibration modes at different rotating speeds. The eigensolutions for a rotating conical and cylindrical shell, annular plate, and ring are compared with existing literature with good agreement. Different shells can be coupled and analyzed using this method.

REFERENCES

[1] Zienkiewicz, O.C. and Taylor, R.L. *The finite element method.* McGraw Hill, Vol. I., (1989), Vol. II., (1991).

[2] Sanders Jr, J.L. Nonlinear theories for thin shells. *Office of Naval Research Technical Report 10* (1961).