Fundamentals of Lax-Wendroff type approaches for hyperbolic problems with discontinuities

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Abstract. The representation of solutions to hyperbolic problems may be traced back to Cauchy-Kowalevski in 1700's in terms of power series and the numerical realization was made by Lax-Wendroff in 1960's. By careful inspection, it is recognized that the Lax-Wendroff approach has irreplaceable values:

(i) It is a unique three-point second order accurate scheme. Any high order scheme should be consistent with the Lax-Wendroff method when it reduces to its second order version. Hence the Lax-Wendroff method is the reference of all high order accurate methods for hyperbolic problems.

(ii) It uses the least stencils (just three points for each time step) and is therefore most compact.

(iii) It is a temporal-spatial coupled method and the useful information of the governing equations are fully incorporated into the scheme. There is no need to exert extra effort even when any other physical or geometrical effects are included.

Nevertheless, the Lax-Wendroff approach just works for smooth flows, and it should be modified to suit for capturing discontinuities. The currently-used generalized Riemann problem (GRP) method is regarded as the discontinuous version of Lax-Wendrof method, and it uses both the Cauchy-Kowalevski methodology and the singularity tracking technique. The resulting scheme is consistent directly with the corresponding physical balance laws in integral form rather than in PDE form.

This lecture will address how fundamental the Lax-Wendroff type approaches are, particularly when solving problems with discontinuities. Keeping the fundamentals in mind, we design multi-stage high order accurate numerical schemes by taking second order Lax-Wendroff type flow solvers as building blocks.