

# ISOGEOMETRIC ANALYSIS FOR 2D AND 3D MHD SUBPROBLEMS: SPECTRAL SYMBOL AND FAST ITERATIVE SOLVERS

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In this talk, we focus on large and highly ill-conditioned linear systems arising from a B-Spline discretization of some 2D and 3D parameter-dependent magnetohydrodynamics (MHD) subproblems encountered when macroscopically describing the behavior of the plasma. We show that the sequence of the involved coefficient matrices belongs to the Generalized Locally Toeplitz (GLT) class [2] and we compute the spectral symbol describing its asymptotic eigenvalue distribution. The study of the symbol and of its eigenvalue functions shows that the coefficient matrices are affected by various sources of ill-conditioning related to both physical and approximation parameters. We use then the retrieved spectral information to design a strategy made up of spectrally complementary iterative solvers able to satisfactorily deal with the sources of ill-conditioning. In detail, we use a 2D and 3D vector extension of the multi-iterative approach already proposed in the literature [1] for the scalar Laplacian operator as preconditioner for a Krylov-type method. Several numerical examples show that the resulting solver is computationally attractive and robust with respect to the relevant parameters.

## REFERENCES

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