A UNIFIED FRAMEWORK FOR GUARANTEED AND ROBUST A POSTERIORI BOUNDS FOR LAPLACE EIGENVALUES AND EIGENVECTORS

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In this talk, we present a general framework for a posteriori error estimates in numerical approximations of the Laplace eigenvalue problem, applicable to all standard numerical methods. Guaranteed and fully computable upper and lower bounds on an arbitrary simple eigenvalue are given, as well as on the energy error in the approximation of the associated eigenvector. The bounds are valid under some gap condition that we will discuss. We give a practical way how to check this assumption as the precision of the resulting estimates depends on these relative gaps.

The bounds feature no unknown (solution-, regularity-, or polynomial-degree-dependent) constant, are optimally convergent (efficient), and polynomial-degree robust. Applications of the framework to nonconforming, discontinuous Galerkin, and mixed finite element approximations of arbitrary polynomial degree are provided, along with some numerical illustrations.

REFERENCES

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