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Mesh-tying via multiple-control PDE-constrained optimization and GMLS function reconstruction

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Separate meshing of subdomains in mesh-tying, transmission, and other interface problems, induces independent interface discretizations that can have gaps and overlaps. This presents a challenge for traditional, Lagrange multiplier formulations because the usual coupling conditions are no longer well-defined over non-coincident interfaces. At the same time, these conditions can be used to define a measure of the solution discrepancy between the spatially non-coincident interfaces. Minimizing this measure, subject to the governing equations on the subdomains, provides an alternative, optimization-based approach for coupling over mismatched interfaces.

In this talk we develop this approach for a model scalar elliptic problem with an interface induced by a discontinuous "material" property. We use continuity of the states and the normal flux to obtain discrepancy measures that define the optimization objective, while restrictions of the governing equations to the discretized subdomains provide the optimization constraints. We close each subdomain equation by specifying a Neumann boundary condition on its respective version of the true interface. These boundary conditions represent the virtual controls driving the minimization process.

We consider two alternatives for implementing the discrepancy measure term involving the normal flux term. In the first approach [1], the flux on the interface is computed by using the gradient of the finite element solution on the associated subdomain. Although this allows us to pass a linear patch test, it also limits the accuracy of the method resulting in optimal H^1, but sub-optimal L^2, convergence rates. The second approach uses a Generalized Moving Least Squares (GMLS) reconstruction [2] to lift the order of the finite element solution before evaluating the normal fluxes, effectively removing the barrier to optimal L^2 convergence. This version of the method utilizes the GMLS implementation in the Compadre toolkit.

We will present the formulation of the method, extensions operators used to compare solutions on non-coincident interfaces, and a GMLS reconstruction operator designed to lift the approximation order of the gradient. Numerical studies will be shown demonstrating optimal convergence of the method for a manufactured solution, successful passing of a patch test for recovering a globally linear solution, and the degree to which global flux is conserved.

REFERENCES

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