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ON THE APPROXIMATION OF THE FULLY DYNAMIC SYSTEM OF FLOW IN DEFORMABLE POROUS MEDIA

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We consider the numerical approximation of the dynamic Biot–Allard equations of poroelasticity

$$\begin{aligned} \rho\partial_t^2\boldsymbol{u} - \nabla\cdot(\boldsymbol{C}:\boldsymbol{\varepsilon}(\boldsymbol{u}) - \alpha\, p\boldsymbol{I}) + \partial_t \int_0^t \boldsymbol{\mathcal{A}}\Big(\frac{(t-\zeta)\eta}{\rho_f l^2}\Big)\cdot\big(\rho_f\boldsymbol{F}(\boldsymbol{x},\zeta) - \nabla p(\boldsymbol{x},\zeta) - \rho_f\partial_\zeta^2\boldsymbol{u}(\boldsymbol{x},\zeta)\big)\,\mathrm{d}\zeta &= \rho\boldsymbol{F}\,, \\ \partial_t\Big(M\,p + \nabla\cdot(\alpha\,\boldsymbol{u})\Big) + \nabla\cdot\Big(\int_0^t \boldsymbol{\mathcal{A}}\Big(\frac{(t-\zeta)\eta}{\rho_f l^2}\Big)\cdot\Big(\boldsymbol{F}(\boldsymbol{x},\zeta) - \rho_f^{-1}\nabla p(\boldsymbol{x},\zeta) - \partial_\zeta^2\boldsymbol{u}(\boldsymbol{x},\zeta)\Big)\,\mathrm{d}\zeta\Big) &= 0\,, \end{aligned} \tag{1}$$

describing fluid infiltration into a porous medium and coupled solid deformation; cf. [4]. Numerous fields of applications in technology and natural sciences would benefit from a better understanding and integration of this problem of fluid-structure interaction. Formally, in the singular limit of vanishing contrast coefficients, the ratios between intrinsic characteristic times and the characteristic time scale of the domain, the quasi-static Biot system (cf. [2]) is obtained from the hyperbolic-parabolic system (1). For larger contrast coefficients, for instance if tiny poroelastic layers are involved, the fully dynamic system (1) needs to be solved.

For this, we discuss different approaches based on space-time finite element discretizations that become feasible for solving the system (1) numerically. Iterative coupling schemes for the quasi-static Biot system (cf. [2]) and monolithic solvers are addressed as building blocks for the approximation of (1). Further, new theoretical results (cf. [1, 3]) for the approximation of the either subproblems in (1) are presented. These results are illustrated by numerical experiments.

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