

# FREE SURFACE CONTROL IN INK-JET PRINTING

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Control strategies for fluid dynamics systems with moving contact lines are of high interest in many applications, ranging from the design of watercraft and marine structures to the microfluidics of sliding droplets and capillary tubes. Indeed, in such systems, the concurrence of a free surface and a solid body and the resulting dynamics of the contact line play a major role in determining the performance of the system under investigation.

The present work, inspired by an application in ink-jet printing, belongs to this framework. The ink inside the printing nozzle is modeled as an incompressible Newtonian fluid, with surface tension and wall friction, and the contact line dynamics is governed by the generalized Navier boundary condition. For the numerical treatment of the differential system, a stabilized arbitrary Lagrangian-Eulerian finite element scheme is adopted [1]. The stabilization is derived from variational principles and effectively damps the spurious oscillations arising from the time-explicit treatment of the geometry.

A control problem is addressed by a computationally highly efficient – albeit suboptimal – instantaneous control technique [2]. Numerical tests show the effectiveness of the proposed procedure in reducing the duration of the transient between the ejection of two subsequent ink jets, by deadening the natural oscillations occurring inside the nozzle. Aiming at further improving the control of the flow system at hand, an actual time minimization problem is addressed, hinging on a time transformation [3]. Preliminary results show the appropriateness of this approach for the problem under investigation, and set promising bases for future work.

## REFERENCES

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