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A ROBUST MIXED FINITE ELEMENT METHOD FOR GENERALIZED POROELASTICITY

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The classical Biot and Terzaghi soil models, describing flow through a single fluid network in a porous and elastic medium, were generalized to equations describing multiple fluid network poroelasticity (MPET) by Barenblatt and Aifantis. The MPET equations have been utilized in geomechanics to simulate multiple fractured strata for a few decades, but are now also beginning to find their application in biomechanics. Indeed, the multiple network poroelasticity theory aptly models the multiple fluid networks encountered in e.g. the brain: such as extracellular spaces, vasculature and paravasculature [1].

In the quasi-static case, these equations read as: for a set of N fluid networks, find the displacement u and the network pressures p_n for $n = 1, \ldots, N$ such that

$$- \operatorname{div} (2\mu\varepsilon(u) + \lambda \operatorname{div} u) + \textstyle\sum_n \alpha_n \nabla p_n = f, \tag{1}$$

$$c_n \partial_t p_n + \alpha_n \operatorname{div} \partial_t u - \operatorname{div} \kappa_n \nabla p_n + S_n = g_n, \tag{2}$$

for $n = 1, \ldots, N$. The characterizing parameters for each fluid network are the Biot-Willis coefficient α_n, the storage coefficient $c_n \geq 0$, and the hydraulic conductivity tensor κ_n, while S_n represent transfer terms in/out of network n. In the context of the brain, the storage coefficients and permeability tensors may typically be very small and with possible jumps. These regimes pose challenges for standard finite element discretizations of the MPET equations. This fact has spurred recent interest in developing numerical methods, and preconditioners, which are robust in the intrinsic limits of elastic, and poroelastic models; including $\lambda \to \infty$, $\kappa_n \to 0$, and $c_n \to 0$.

In this work, we propose a new mixed finite element formulation for the multiple-network poroelasticity equations.The key idea is to introduce the network fluid fluxes as additional variables

$$z_n = -\kappa_n \nabla p_n. \tag{3}$$

targeting a finite element formulation that is robust with respect to low hydraulic conductivities κ_n and storage coefficients c_n. We will present both theoretical and numerical results on the robustness and convergence of the new method, together with numerical demonstrations relating to the topic of cerebral interstitial and paravascular fluid flow.

REFERENCES

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