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WILL ENTROPY STABILITY LEAD TO CONVERGENT NUMERICAL SCHEMES?

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Ever since Lax and Richtmyer's convergence theorem was proved in the 50's, it has been known that stability and consistency lead to (grid-)convergent numerical schemes for linear PDEs. However, the flow equations are not linear and the theorem does not apply. (If the solution is smooth, an argument can be made that linear stability implies convergence, but that is not the current topic.)

Nevertheless, stability, in some appropriate sense, will still be necessary to ultimately prove convergence for the non-linear flow equations. In recent years, entropy stability ([1]) has emerged as a popular choice to obtain non-linear stability in CFD. The idea is to design numerical schemes that satisfy a local entropy inequality. With certain assumptions, the entropy estimate can be recast as L^2 estimates of the conservative variables. In [2], an entropy stable and kinetic-energy stable scheme for the compressible Navier-Stokes equations was analyzed. Under various assumptions, it was shown that the continuity and momentum equations were weakly convergent but the energy equation was beyond reach.

In this talk, we will discuss the assumptions associated with entropy stability and hence its possibilities and limitations with respect to convergence. Moreover, we will also discuss what other estimates a CFD scheme should satisfy to be convergent and how convergence is affected by the complexity of the scheme itself. The following questions are central:

- To what extent is entropy stability useful?
- Assume that a scheme A satisfies stronger estimates than a scheme B. Can scheme A be non-convergent while scheme B is convergent?
- Can a high-order scheme be convergent for a general problem?

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