

# Efficient “Matrix-Free” Partial-Assembly Algorithms for High-Order ALE Hydrodynamics

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BLAST is a multi-material arbitrary Lagrangian-Eulerian (ALE) code based on high-order finite elements [1]. The core of BLAST is the Lagrange phase where we solve the time-dependent Euler equations of compressible gas dynamics on a moving mesh with explicit high-order time-stepping. This algorithm requires the solution of a global linear system for solving the momentum conservation equation, involving both a “mass” and a rectangular “force” matrix. In our standard solution approach, we fully assemble global, sparse, parallel distributed matrices for the momentum solve. This approach has poor scaling characteristics as a function of the polynomial degree of the finite elements used for discretization. In addition, solution of the global linear system for momentum requires parallel matrix-vector multiplies which is a bandwidth limited computational kernel.

In this talk, we discuss our recent work in converting the Lagrange phase of BLAST to use more efficient partial assembly methods where instead of assembling and storing a global mass and force matrix, we store only data at quadrature points (requiring substantially less data transfer) and decompose the assembly process into a set of element wise operator evaluations (i.e. computing the action of the operator per element) which reduces the problem to performing element wise dense tensor contractions. At higher finite element orders, this results in a compute bound algorithm that is well suited for GPU acceleration. In addition, we can exploit the tensor product nature of finite element basis functions on quadrilateral/hexahedral elements leading to a reduction in required operations. To explore the properties of this approach, we have developed a miniapp, called LAGHOS [2], which captures the key properties of the Lagrange phase of BLAST. LAGHOS supports both full and partial assembly methods. We report on performance characteristics of both LAGHOS and BLAST using the partial assembly approach and discuss the code changes that were required to enable this.

## REFERENCES

- [1] V. Dobrev, T. Kolev, R. Rieben, “High order curvilinear finite element methods for Lagrangian hydrodynamics,” *SIAM J. Sci. Comp.*, 34(5), P606-641, 2012.
- [2] LAGHOS Miniapp: <https://github.com/CEED/Laghos>

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