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# High Order Discontinuous Galerkin Method with Spalart–allmaras Turbulence Model for Hypersonic Flow Simulations

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The objective of the present study is to investigate high order discontinuous Galerkin (DG) methods for the simulation of hypersonic turbulent flows. In particular, we are interested in the DG discretization of the Reynolds-averaged Navier-Stokes (RANS) equations. While the previous work involving DG with turbulence model has been mainly focused on non-smooth behaviour of turbulence working variables on the interface between turbulent/non-turbulent regions and most simulations have been conducted for subsonic and transonic simulations, this study presents high order DG method with the SA model to the application of hypersonic turbulent simulations. An artificial viscosity approach based on two different types of sensors is proposed as shock-stabilization techniques and other issues including implicit time discretization, large aspect ratio meshes and assessment of turbulence models are also addressed in the present work.

The suggested artificial viscosity approach builds on the formulation proposed in [1] where the artificial viscosity coefficient $\varepsilon$ is determined according to the oscillation sensor $S_K$ that can be evaluated as

$$S_K = \frac{\left(q_h - \overline{q}_h, q_h - \overline{q}_h\right)_K}{\left(q_h, q_h\right)_K} \tag{1}$$

where $q$ and $\overline{q}$ respectively represent the complete (order $P$) and truncated (retaining modes up to order $P-1$) expansions of a convenient property (here we use averaged Mach number). Necessary modification is made in the current work when extending this artificial viscosity model to hypersonic RANS simulations. First, the methods include anisotropic mesh size metrics to obtain more robust behaviour on anisotropic viscous meshes. Second, apart from using the resolution sensor in Eq. (1) we also adopt an entropy residual sensor expressed as [2]

$$E_K = \frac{1}{|\Omega|}\int_\Omega \left[\frac{\partial \Psi(U)}{\partial t} + \nabla \bullet F(U)\right] d\Omega \tag{2}$$

where $\Psi=\rho s$ indicates the entropy function and $F$ represents the corresponding entropy flux. Different from the published methods, here we employ the entropy residual sensor to remark the cells that the oscillation sensor has identified and adjust the value of the artificial viscosity coefficient. Hence the resulting approaches contain two sensors from both Eq. (1) and Eq. (2). Furthermore the artificial viscosity is implemented within fully implicit discretization of a $p$-multigrid preconditioned GMRES solver with line-implicit linearized Gauss-Seidel as single grid smoother that is efficient for those large aspect-ratio cells. The full Jacobian matrices are evaluated by numerical difference considering both the artificial viscosity term and turbulence model term.

Preliminary results of a hypersonic turbulent plate at $M_\infty$=6, $Re$/inch=2.6×$10^6$/inch are displayed in Figs. 1-2. We see a wider extent and a smaller value of artificial viscosity obtained by the present method using both sensors. The results of velocity profiles and wall skin friction show better agreement with DNS solution for the higher order schemes. However a much higher level of eddy viscosity is observed for the current SA turbulence model, which is also shown in the simulation of turbulent compression corners in Fig. 3. The higher eddy viscosity may be caused by the direct extension of SA model to the hypersonic flows and in the final paper we will carry out an assessment of the compressible correction and the choice of model coefficients for the current SA implementation in the hypersonic flow simulation.

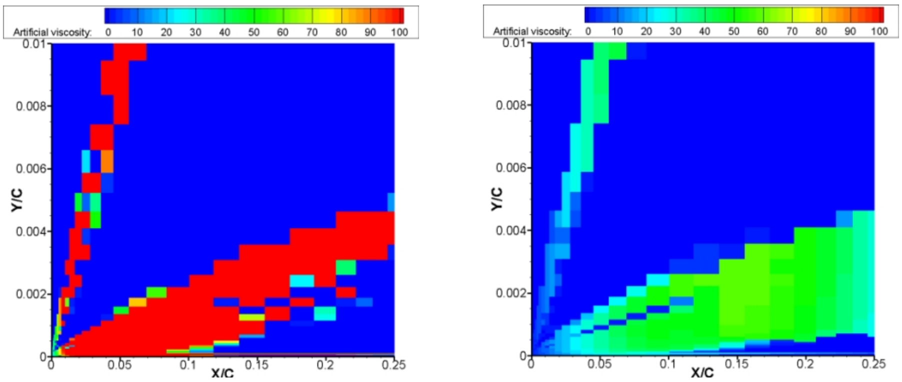



Fig. 1  Comparison of artificial viscosity obtained by the original method (left) and present method (right) for the hypersonic turbulent plate at $M_\infty$=6, $Re$/inch=2.6×10$^6$/inch.

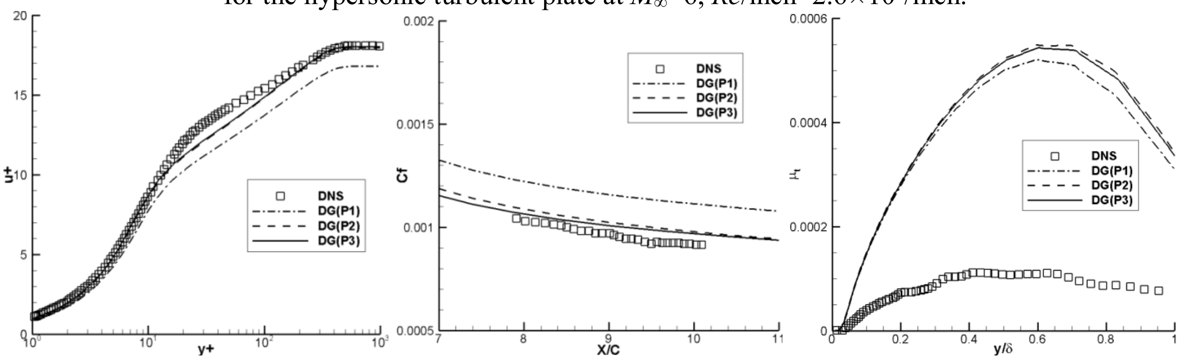
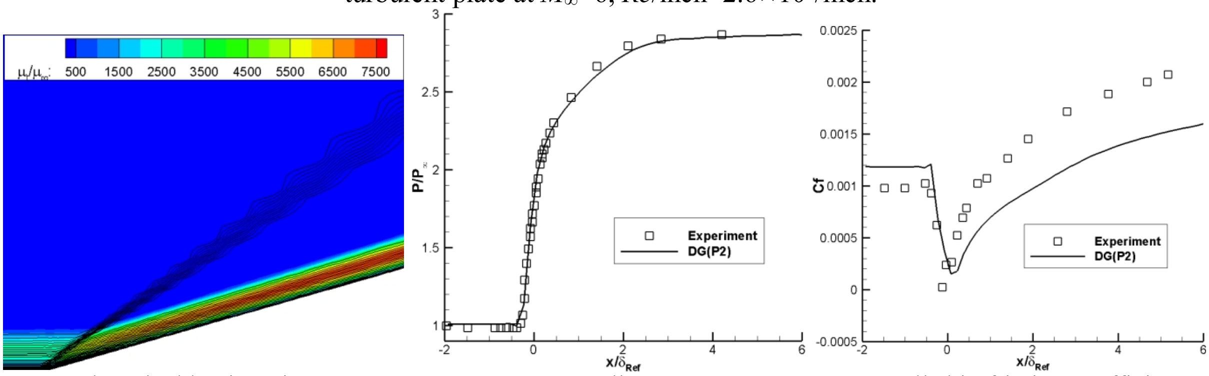



Fig. 2  Distribution of velocity profiles, wall skin friction coefficient and eddy viscosity for the hypersonic turbulent plate at $M_\infty$=6, $Re$/inch=2.6×10$^6$/inch.




**a** Mach and eddy viscosity counters **b** Wall pressure **c** Wall skin friction coefficient

Fig. 3  Results for 16° turbulent compression corners at $M_\infty$=2.85, $Re_\delta$=1.71×10$^6$.

## REFERENCES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