6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11 15 June 2018, Glasgow, UK

# HYBRIDIZED SEMI-IMPLICIT SOLVERS FOR COMPATIBLE FINITE ELEMENT METHODS

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**Key words:** *Compatible finite element methods, numerical weather prediction, hybridization*

Compatible finite element methods have been proposed for numerical weather prediction in [1, 2] as discretisations that extend the properties of the staggered C-grid finite difference method to arbitrary meshes as well as to higher-order representations. One of the key differences with the C-grid method is that the mass matrix for the velocity is no longer diagonal, which makes it difficult to use formation of a reduced Helmholtz-type equation for the pressure increment in a semi-implicit solver. For classical elliptic problems, a technique known as hybridization has been used for decades in the context of compatible finite element discretisations (see [3] for a summary). In this technique, the continuity conditions on the velocity space are removed, only to be restored by enforcing them using Lagrange multipliers that are supported as trace functions on cell interfaces. The (now discontinuous) velocity and pressure can be eliminated in each cell, leading to a sparse elliptic operator for the Lagrange multipliers which can be solved by standard techniques such as algebraic multigrid methods with SOR smoothers. In this talk we will describe how this approach may be applied to the linear system that arises in semi-implicit discretisation of the compressible rotating Euler equations in three dimensions. The implementation of these systems is challenging, and we will introduce a code-generation system (Slate) that addresses the assembly and solution of hybridizable systems within the Firedrake framework. We will also present our latest results demonstrating the performance of semi-implicit solvers using this technology within the Gusto compatible finite element dynamical core toolkit.

## REFERENCES

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