

Upscaling mass transfer in brain capillary networks

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Brain perfusion imaging techniques rely on the measurement of spatio-temporal concentration fields of various endogenous or exogenous tracers in the brain tissue.

Their resolution is typically between 1 mm^3 (Magnetic Resonance Imaging) and $(10 \text{ mm})^3$ (Positron Emission Tomography). This is much coarser than the diameters of most arterioles and venules, which are typically below $100 \text{ }\mu\text{m}$, and, of course, of capillaries, whose diameters are tenfold smaller. This implies that methods to deduce the regional blood flow rate out of these large-scale concentration fields should rely on upscaled models, i.e. models describing the macro-scale behavior of the vascular system with effective properties taking into account its microstructure.

To derive such models, the Volume Averaging Technique, which has been previously developed for upscaling mass transfer in heterogeneous porous media [1], can be applied to the advection-diffusion equations. Capillary networks indeed exhibit a space filling mesh-like structure [2], for which a Representative Elementary Volume (REV), can be extracted: a 3D network of capillaries with diameters ranging from 1 to $10 \text{ }\mu\text{m}$ embedded in tissue, with volume about $(150 \text{ to } 300 \text{ }\mu\text{m})^3$. In this technique, closure equations must be solved in REVs to deduce effective coefficients, representing its macro-scale behavior.

Being able to solve closure equations on any 3D network geometry taking into account individual vessels is a computational challenge. Here, we developed a numerical framework to solve partial differential equations on anatomically accurate capillary networks using the finite element library Feel++ [3]. This framework is used to 1) solve the closure equations on a REV and deduce its effective coefficients and 2) perform direct simulations of mass transfers problems as references to validate the upscaling procedure.

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REFERENCES

- [1] Quintard et al, *International Journal of Heat and Mass Transfer*, 38:2779-2796, 1995.
- [2] Lorthois et al, *Journal of Theoretical Biology*, 262:614-633, 2010.
- [3] Prud'homme et al, *ESAIM: Proc.* 38:429-455, 2012.