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NUMERICAL ASPECTS OF THE FRACTIONAL STURM-LIOUVILLE PROBLEM

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We consider the fractional differential equation called the fractional Sturm-Liouville equation (FSLE) [1]

$$ {}^{C}D^{\alpha}_{b^-}\left(p(x)\,{}^{C}D^{\alpha}_{a^+}y(x)\right)+q(x)y(x)=\lambda w(x)y(x) \tag{1}$$

subjected to the mixed boundary conditions

$$ y(a)=0,\quad {}^{C}D^{\alpha}_{a^+}y(x)\Big|_{x=b}=0 \tag{2}$$

In this paper, we investigate numerically the eigenvalues and eigenfunctions associated to the FSLP (1)-(2) [2]. We present two numerical approaches. The first one is based on discretization of the left and right fractional derivatives by utilizing the finite difference method. In the second approach we transform the fractional differential problem to the equivalent integral one and then we discretize the fractional integral operators.

Both introduced numerical schemes lead to the set of eigenvalues and to an orthogonal system of approximate solutions. The experimental order of convergence (EOC), both for eigenvalues and eigenfunctions, is analysed.

The numerical approaches, presented in this paper, for finding the eigenvalues and eigenfunctions of FSLP can be used to get the approximate solution to the fractional diffusion problem in a finite domain [3].

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