

High Order Advection of Magnetic Fields for ALE Magnetohydrodynamics

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We are interested in the computational simulation of magnetohydrodynamic problems using a higher order unstructured grid finite element method. The problems of interest are primarily pulse power problems, these problems typically involve materials that are initially solid but due to extreme pressures and temperatures will melt and flow and even vaporize to form a plasma. These problems are driven by electric currents and magnetic fields. Because the boundaries between conducting and insulating materials are important, the problem is formulated using an arbitrary Lagrangian-Eulerian method to preserve material interfaces for as long as possible.

The hydrodynamic equations are solved via a Lagrange plus Remap (indirect ALE) formalism using high order curvilinear finite elements [1][2] with a high order remap step [3] applied when the computational mesh is relaxed. The magnetic diffusion equation is solved implicitly (operator split from hydrodynamics) in the Lagrangian frame using a higher order mixed finite element method with H(div) functions for the magnetic field and H(curl) functions for the electric field [4]. Similarly to the hydrodynamics, the magnetic field is advected in pseudo-time during the remap step. The key challenge of the magnetic advection step is the desire to preserve the divergence-free character of the magnetic field. Two methods are investigated for the advection of the magnetic field, an L2 projection method that employs an intermediate H(curl) field (the induced EMF) and is exactly divergence preserving, and a discontinuous Galerkin method using only the H(div) magnetic field. Both advection methods are implemented using arbitrary order spatial discretization and 4th order explicit time integration.

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