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On Numerical Integrations of Discontinuous Galerkin Methods over High-order Curved Elements

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The high-order methods are emerging as the latest trend in computational fluid dynamics (CFD) community to simulate more complex flow problems with better resolution and less numerical damping. It, however, still faces a few obstacles such as shock-driven instabilities and severe computational costs, which challenges their potentiality. Among them, significant amount of computations mainly keeps both academia and industry from applying the high-order methods to practical purposes in real-life problems. In discontinuous Galerkin (DG) methods, efficient handling of numerical integrations is a crucial component that decides its numerical accuracy and efficiency. Conventional approaches to integrate these terms are based on the prescribed quadrature rules that are iteratively calculated and optimized in the reference domain. Its computing costs, however, exponentially increase when using high-order elements because of its coordinate transformation to the reference domain. In the case of three dimensions, the number of quadrature point for the domain integration is proportional to the cubic order for both solution approximation and elements, and, on top of that, it is required to evaluate state variables and flux functions at each point. This greatly increases the amount of computations in DG methods during runtime. In order to resolve this issue, advanced numerical approaches to integrate the boundary and domain integral terms of DG methods are invented to significantly reduce computational costs. In this study, a novel approach to perform direct integration on the physical domain is proposed. The proposed method depends only on the order of solution approximation, not on the order of elements, so it shows remarkable computational efficiency on high-order elements. Although positive quadrature weights are not guaranteed, it requires no iterative optimization and its quadrature points and weights can be readily computed during pre-processing. In addition, one more approach to reduce the number of solution and flux evaluations during numerical integration is provided. The flux is directly reconstructed and integrated using a relatively small number of points compared with conventional methods. In order to do this, orthonormal polynomials are generated by the modified Gram-Schmidt process under the specified inner product space. The flux functions are then reconstructed by Lagrange polynomials that are computed from the orthonormal bases. Except for the boundary integration in two dimensions, the new approaches dramatically reduce the amount of computations by diminishing the number of solution and flux evaluations. For example, for the DG-*P3* method on three-dimensional *P3*-meshes, the amount of evaluations is reduced by up to 20 times for the domain integration and 3 times for the boundary integration. Computational efficiency dramatically improves even more as the order of mesh and/or solution approximation increases.