

# Time Discretization Error Analysis in Conventional and Multifield Plasticity

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In order to optimize mass products' properties, alternative fabrication procedures are developed. As an example an integrated thermomechanical forming process of a flange shaft can be considered, cf. [6]. One of the main topics of this manufacturing process is characterized by plastic deformations and dynamical effects which cannot be neglected. Thus, adequate material models as well as proper numerical schemes have to be established. As a first step in that direction the implementation of higher order accurate time integration schemes in the regime of small strain ideal isotropic elastoplasticity in combination with appropriate error estimators is investigated.

In this context a conventional approach towards elastoplasticity which is based on the radial return map is presented. Therein, the balance of linear momentum is discretized using the finite element method and the material laws are evaluated on GAUSS point level, cf. [5]. Thus, a divided time discretization method is the consequence.

In contrast, a variational approach is shown. Therein, a dissipation potential is assumed and the principle of JOURDAIN is exploited to generate a multifield formulation of plasticity enabling the usage of one single time discretization scheme on element level, cf. [3,4]. Hence, the application of higher order time integration methods to both the evolution equation and to the balance law is simplified, cf. [1,2]. As a drawback the number of unknowns is increased and a semismooth NEWTON method judging whether elastic or plastic phenomena prevail has to be elaborated.

A comparison between the conventional and the variational procedure demonstrates the differences between both schemes, focusing on the time discretization error using diagonally implicit RUNGE-KUTTA methods.

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