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NONSMOOTH MULTIGRID METHODS FOR PHASEFIELD FRACTURE FORMATION PROBLEMS

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Many phasefield formulations of fracture formation problems are written as gradient flows of an energy functional. Increment problems of such flows are typically nonsmooth minimization problems, for energies that are convex or at least close to it. Such problems are typically solved using predictor-corrector or operator-splitting methods. These are expensive, because they solve sequences of linear problems. Also, their convergence behavior is not always clear.

We propose a nonsmooth multigrid method that can solve such problems roughly in the time of one equivalent linear problem. This is shown experimentally for the brittle fracture model of [2], where the increment problems are biconvex minimization problems. The method has been shown to converge to a stationary point for any initial iterate [1].

As a step towards ductile fracture we apply the same algorithm to a model of small-strain rate-independent plasticity. For such a model, the primal formulation again leads to a sequence of convex minimization problems. The nonsmooth multigrid method applied to these minimization problems converges in roughly the time of a single predictor-corrector iteration, while remaining provably convergent [3].

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