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Application of symbolic and automatic differentiation frameworks to multi-physics problems

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Magnetorheological elastomers are smart, field-responsive composite materials that are of increasing interest in numerous industries, with much work being conducted to characterise [1] them. A similar growth in research is notable among other fields involving coupled media and multi-physics problems in general. As the complexity of the physics describing the material loading [2] and response increases, so does the time cost and sophistication of the computational tools used to analyse these problem both in the academic and industrial settings. Associated with this are further challenges, one example being that potential errors in implementation and subsequent numerical instabilities could be misinterpreted as physically plausible material instabilities.

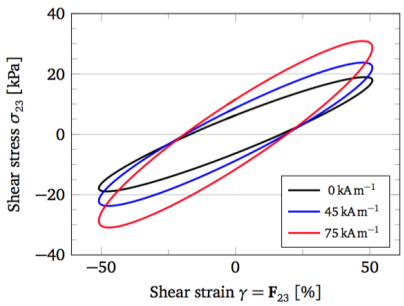

Fig. 1 Coupled rate-dependent constitutive law

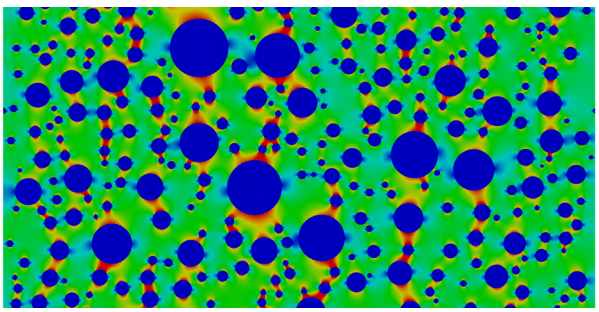

Fig. 2 Microstructural analysis of coupled media using the Finite Element Method

Tools that automate differentiation are one mechanism to tackle the challenges of constitutive law and finite element formulation prototyping, development and validation. In this work we discuss the implementation of frameworks to perform assisted differentiation within the deal.II finite element library [3,4]. We outline and discuss some of the challenges surrounding the integration of two automatic [5,6,7] and symbolic [8] differentiation libraries within the pre-established tensor and finite element frameworks. Lastly, we demonstrate and benchmark the current implementation in the context of two problems, namely (i) a rate-dependent magnetic field-response constitutive law and (ii) homogenisation of representative volume

element for a rate-independent magneto-rheological elastomer.

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