Model Reduction for Multiple Transport Phenomena with Non-Periodic Boundary Conditions

P. Schulze[1], J. Reiss[2], and V. Mehrmann[1]

[1] Institute of Mathematics, Technische Universität Berlin
Str. des 17. Juni 136, 10623 Berlin, Germany
{pschulze, mehrmann}@math.tu-berlin.de, http://www.tu-berlin.de/?id=66282
[2] Institute of Fluid Dynamics and Technical Acoustics, Technische Universität Berlin
Müller-Breslau-Straße 15, 10623 Berlin, Germany
reiss@tnt.tu-berlin.de, http://www.cfd.tu-berlin.de/~reiss/

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In the last decades, model order reduction (MOR) schemes have become a useful tool for reducing the computational effort of applications where a model needs to be simulated multiple times for different parameter values or inputs, e. g., optimization and control. For model order reduction of nonlinear systems, a commonly used method is the proper orthogonal decomposition (POD) also known as principal component analysis or Karhunen Loeve Expansion. The POD has shown to be a very efficient model reduction tool in many applications, but for systems where a structure with a high gradient, like a shock wave, is transported, neither the POD nor other standard MOR methods are typically capable of reducing the number of unknowns to a great extent.

Currently, there are only few results regarding model reduction of such transport-dominated phenomena, especially for the case of multiple transport directions. A new approach introduced in [1] is the shifted POD (sPOD), which is a mode decomposition scheme capable of low-dimensional descriptions of multiple transport phenomena by introducing time-dependent shifts of the modes. The algorithm proposed in [1] is heuristic but outperforms the classical POD when considering transport-dominated phenomena. The work in [1] mostly focuses on periodic boundary conditions but does not investigate the case of non-periodic boundary conditions thoroughly.

In this talk we present an extension of the sPOD to the case of non-periodic boundary conditions. The key ingredients of the new approach are a reformulation of the sPOD as an optimization problem and a suitable definition of the shift operator in the non-periodic case. This extension of the sPOD allows to construct low-dimensional decompositions of advection-dominated systems with non-periodic boundary conditions. We demonstrate the new algorithm by means of numerical examples including the one-dimensional linear wave equation with outlet and with reflecting boundary conditions. For the considered test cases, the sPOD yields mode decompositions with far fewer modes than the classical POD with comparable accuracy and appears to be well-suited for structure identification in advection-dominated systems.

REFERENCES

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