

Nucleation and Propagation of Fracture and Healing in Elastomers: A Phase-Transition Theory & Numerical Implementation

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A macroscopic theory is proposed to describe, explain, and predict the nucleation and propagation of fracture and healing in elastomers undergoing arbitrarily large quasistatic deformations [1]. The theory, which can be viewed as a natural generalization of the phase-field approximation of the variational theory of brittle fracture of Francfort and Margio [2] to account for physical attributes innate to elastomers that have been recently unveiled by experiments at high spatio-temporal resolution [3,4], rests on two central ideas. The first one is to view elastomers as solids capable to undergo finite elastic deformations and capable also to phase transition to another solid of vanishingly small stiffness: the forward phase transition serves to model the nucleation and propagation of fracture while the reverse phase transition models the possible healing. The second central idea is to take the phase transition to be driven by the competition between a combination of strain energy and hydrostatic stress concentration in the *bulk* and surface energy on the created/healed new *surfaces* in the elastomer.

From an applications point of view, the proposed theory amounts to solving a system of two coupled and nonlinear PDEs for the deformation field and an order parameter, or phase field. A numerical scheme is presented to generate solutions for these PDEs in $N = 2$ and 3 space dimensions. This is based on an efficient non-conforming finite-element discretization, which remains stable for arbitrarily large deformations and elastomers of any compressibility, together with an implicit gradient flow solver, which is able to deal with the large changes in the deformation field that can ensue locally in space and time from the nucleation of fracture. We conclude by presenting sample simulations of the so-called Gent-Park experiment and confronting those with recent experimental results for various types of silicone elastomers.

REFERENCES

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