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Reduced Basis method applied to a convective instability Rayleigh-Bénard problem

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The reduced basis approximation is a discretization method that can be implemented for solving of parameter-dependent problems $P(\phi(\mu); \mu) = \mathbf{0}$ with parameter μ in cases of many queries. This method consists of approximating the solution $\phi(\mu)$ by a linear combination of appropriate preliminary computed solutions $\phi(\mu_k)$ with $k = 1, 2, \ldots, N$ such that μ_k are parameters chosen by an iterative procedure using the kolmogorov n-width measures [2, 4].

In [1], the reduced basis method is applied to a two dimensional incompressible Navier-Stokes equations with constant viscosity and the Boussinesq approximation coupled with a heat equation that depends on the Rayleigh number, $P(\phi(R); R) = \mathbf{0}$.

Rayleigh-Bénard convection problem displays multiple steady solutions and bifurcations by varying the Rayleigh number, therefore the eigenvalue problem of the corresponding linear stability analysis has to be implemented. A linear stability analysis of these solutions is performed in [3] by a spectral collocation method.

In this work the eigenvalue problem of the corresponding linear stability analysis is solved with the reduced basis method. It is considered the aspect ratio $\Gamma = 3{,}495$ and R varies in [1000; 2000] where different stable and unstable bifurcartion branches appear [1, 3]. The reduced basis method is suitable to catch bifurcations and instabilities of stationary solutions for many values of R. The reduced basis considered belong to the eigenfunction spaces coming from the eigenvalue problems for different types of solutions in the bifurcation diagram. The eigenvalues and eigenfunctions are easily calculated and the bifurcation points are exactly captured. The resulting matrices are small and this allows a drastic reduction of the computational cost on the eigenvalue problems.

The problem is numerically solved by the Galerkin variational formulation using the Legendre Gauss-Lobatto quadrature formulas together with the reduced basis $\{ \Psi(R_k), \ k = 1, 2, \ldots, N \}$ such that $\Psi(R)$ can be expressed as a linear combination of elements of the reduced basis $\Psi(R_k)$; $k = 1, 2, \ldots, N$.

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