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Mathematical modeling and numerical simulation of magneto-elastic coupling

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Magnetoelastic coupling describes the mutual dependence of the elastic and magnetic fields and can be observed in certain types of materials, among which are the so-called *magnetostrictive materials*. They belong to the large class of *smart materials*, which change their shape, dimensions or material properties under the influence of an external field. The coupling of mechanical and electromagnetic fields is particularly observed in *giant magnetostrictive materials*, alloys of ferromagnetic materials that can exhibit several thousand times greater magnitudes of magnetostriction than the common magnetostrictive materials. These materials have wide applications areas, including robotics, vibration control, hydraulics and sonar systems.

Although the computational treatment of coupled problems has seen great advances over the last decade, the underlying problem structure is often not fully understood nor taken into account when using black box simulation codes. A thorough analysis of the properties of coupled systems is thus an important task.

The talk focuses on the mathematical modeling and analysis of the coupling effects in magnetostrictive materials. Under the assumption of linear and reversible material behavior with no magnetic hysteresis effects, a coupled magnetoelastic problem is set up using two different approaches: the magnetic scalar potential and vector potential formulations. On the basis of a minimum energy principle, a system of partial differential equations is derived and analyzed for both approaches.

The distinctive feature of this work is the analysis of the obtained coupled magnetoelastic problems with regard to their structure, strong and weak formulations, the corresponding function spaces and the existence and uniqueness of the solutions. We show that the model based on the magnetic scalar potential constitutes a coupled saddle point problem with a penalty term. The main focus in proving the unique solvability of this problem lies on the verification of an inf-sup condition in the continuous and discrete cases. Furthermore, we discuss the impact of the reformulation of the coupled constitutive equations on the structure of the coupled problem and show that in contrast to the scalar potential approach, the vector potential formulation yields a symmetric system of PDEs.

The presented models are supplemented with numerical simulations carried out with MATLAB for a 2D magnetostrictive plate in the state of plane stress. The simulations are based on material data of Terfenol-D, a giant magnetostrictive materials used in many industrial applications.