Multigrid method for Stokes problem with discontinuous viscosity

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The goal of presented work is to develop robust scalable solver for linear system arising from finite element discretization of Stokes problem with strongly variable visosity. We consider highly variable, subdomain-wise constant viscosity. Such systems appear in fluid-structure problems[4].

We apply multilevel method developed by Breass and Sarazin[3] and further refined by Zulehner[2]. A multigrid algorithm works on saddle point problem by applying constrained smoother. The relaxation allows iterative procedure to remain in divergence free subspace while smoothing out error in both pressure and velocity.

We consider multilevel methods as preconditioners for Krylov subspace methods. For elliptic problems multilevel methods combined with CG exhibits robustness with respect to variable coefficients. Here, we demonstrate similar performance in case of multigrid preconditioned GMRes by numerical experiments. Similarly to other multilevel methods, the algorithm presented here does not require explicitly storing the matrix. We present here matrix-free implementation using *deal.II*[1] library.

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