

Manifold basis functions with sharp features for isogeometric analysis on unstructured meshes

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Manifold-based surface construction techniques are well known in geometric modelling and a number of variants exist. Common to all is the concept of constructing a smooth surface by blending together overlapping charts as in differential geometry description of manifolds. We combine manifold techniques with conformal parameterisations and the partition-of-unity method to derive basis functions on unstructured quadrilateral meshes.

The obtained basis functions correspond to the vertices of the mesh and have arbitrary prescribed smoothness and approximation order. Each chart on the manifold consists of several elements and has a corresponding planar chart with a smooth one-to-one mapping onto the manifold. On the collection of conformally parameterised planar charts the partition-of-unity method is used for approximation. The smooth partition-of-unity, or blending, functions are assembled from tensor-product b-spline segments defined on a unit square. Polynomials with prescribed degree and continuity are used as local approximants on each chart. Sharp features are represented with suitably chosen C^0 -continuous local polynomials. As will be demonstrated, this sharp basis functions have to be carefully constructed in order to be suitable for both geometric modelling and analysis. This is achieved by considering a sequence of affine and conformal mappings depending on the local connectivity of the mesh and the arrangement of sharp features. To this end, new k -charts, where k is the number of crease edges, are introduced by conformally mapping tensor-product Lagrangian polynomials defined on a unit square.

Our numerical simulations indicate the optimal convergence of the resulting approximation scheme for Poisson problems and near optimal convergence for thin-plate and thin-shell problems discretised with structured and unstructured quadrilateral meshes.

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