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EFFICIENT SOLVERS AND TENSOR-PRODUCT PRECONDITIONERS FOR THE IMPLICIT TIME INTEGRATION OF DISCONTINUOUS GALERKIN METHODS

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Implicit time integration for discontinuous Galerkin (DG) discretizations is important in the context of boundary layer flows, anisotropic, unstructured meshes, and high degree polynomial approximations. Effective preconditioning strategies are essential to the efficient iterative solution of the resulting large, sparse linear systems. This work concerns two topics: (1) fully-implicit Runge-Kutta solvers, and (2) tensor-product preconditioners for very high polynomial degrees.

(1) There are several advantages to using fully-coupled implicit Runge-Kutta schemes compared with traditional DIRK or BDF methods. However, such methods couple all of the Runge-Kutta stages, resulting in a much larger system of equations. We transform the resulting system of equations to maximize sparsity, and then develop several ILU-based preconditioners with favorable performance properties. These solvers have the additional advantage that they allow for parallelism across the stages.

(2) Furthermore, the DG method allows for arbitrary order of accuracy, according to the degree of polynomial approximation used. High-degree polynomials result in extremely restrictive CFL conditions, motivating the use of implicit solvers. We develop efficient solvers and preconditioners that exploit the natural tensor-product structure of quadrilateral and hexahedral grids in order to obtain methods with optimal computational complexity.

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