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Truncated nonsmooth Newton multigrid for nonsmooth minimization problems

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Many problems originating from continuum mechanics and material science lead to large scale nonsmooth minimization problems after discretization in time and space. Examples are classical binary or multi-component phase field models for phase transition and separation, frictional contact problems, plasticity, and phase field-like approaches for brittle fracture. In all these problems nonsmoothness arises either from constraints or from non-differentiable potentials. Hence classical minimization methods like, e.g., Newton's method, are not applicable.

We present the Truncated Nonsmooth Newton Multigrid (TNNMG) method for nonsmooth minimization problem which combines a relaxation approach with nonsmooth Newton and linear multigrid techniques. The TNNMG methods is applicable to a wide range of problems including convex and non-convex minimization. It exhibits optimal complexity and, in many cases, mesh independent convergence rates [3, 2, 1, 4]. This leads to an overall efficiency comparable to linear multigrid, i.e., solving one nonsmooth minimization problem with TNNMG is essentially as expensive as solving one linear problem with multigrid, one of the most efficient methods know. In the talk we will introduce the algorithm, discuss convergence, and present some numerical examples illustrating the efficiency of TNNMG.

REFERENCES

[1] C. Gräser and R. Kornhuber. Multigrid methods for obstacle problems. *J. Comp. Math.*, 27(1):1–44, 2009.

[2] C. Gräser and O. Sander. Polyhedral Gauß–Seidel converges. *J. Numer. Math.*, 22(3):221–254, 2014.

[3] C. Gräser and O. Sander. Truncated nonsmooth Newton multigrid methods for simplex-constrained minimization problems. Preprint 384, IGPM Aachen, 2014.

[4] C. Gräser and O. Sander. Truncated nonsmooth Newton multigrid methods for block-separable minimization problems. Preprint, 2017. arxiv:1709.04992.