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A stabilized cut discontinuous Galerkin framework for mixed dimensional coupled problems

André Massing[1] **and Ceren Gürkan**[1]

[1] Department of Mathematics and Mathematical Statistics,
Umeå University, SE-90187 Umeå, Sweden, {andre.massing,ceren.gurkan}@umu.se

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We develop both theoretically and practically a novel cut Discontinuous Galerkin framework (cutDG) by combining stabilization techniques from the cut finite element method [1] with the classical interior penalty discontinuous Galerkin methods (DG) for elliptic [2] and hyperbolic problems [3]. The resulting framework allows for the unified numerical treatment of a wide range of problems, including boundary and interface problems as well as surface and multidimensional, coupled surface-bulk and interface-bulk problems.

The key idea is that the domains of interest such the surface or the bulk domain can be embedded into a background mesh in an unfitted manner. Using only a few abstract assumptions on the employed cutDG stabilization, we can establish geometrically robust optimal a priori error and condition number estimates irrespective of how the embedded geometry cuts the background mesh. Possible realizations of the cutDG stabilization are discussed. Motivated by flow and transport problems in fractured porous media, the theoretical properties are corroborated by a number of elliptic and hyperbolic prototype problems ranging from simple boundary values problems to multidimensional coupled interface-bulk problems.

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