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TAIL FRAGILITY AS A TOOL FOR MODEL CONFIDENCE

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Most computational models, that is, the combination of a mathematical model and a discretization scheme, are judged based on their performance in nominal cases. In terms of numerical methods the measures used might be convergence rates, stability, etc., and in mathematical modeling performance may be based on the accuracy of the model in known physical regimes.

In this presentation we adapt a measure of fragility developed for financial risk by Taleb and Douady [1] and explained in detail in [2] based on the sensitivity of the tails of a distribution to parameters. Our adaptation looks at the probability of failure rather than cost of failure required in financial models. In particular we desire to look at the probability of a random variable X exceeding some threshold value x_U. Call this probability the probability of failure, P_{fail}. If the distribution of X depends on some parameter λ we can write

$$P_{\text{fail}} = 1 - F(x_U; \lambda), \tag{1}$$

where $F(x;\lambda)$ is the CDF of X. Then the absolute sensitivity to the probability of failure to the parameter λ, $V(x_U, \lambda)$, is

$$V(x_U, \lambda) = \left| \frac{\partial F}{\partial \lambda}(x_U; \lambda) \right|. \tag{2}$$

Implicit in this definition is that the larger the value of $V(x_U, \lambda)$, the larger the impact on the probability of failure and the more scrutiny conclusions about the probability of failure should undergo.

To apply this metric to scientific simulation we consider a quantity of interest from a calculation, Q that is the function of many parameters such as numerical (mesh) parameters ($\vec{\lambda}_m$), model parameters ($\vec{\lambda}_p$), and random, aleatoric uncertainties ($\vec{\lambda}_r$). We could rightly ask what is the sensitivity of the model prediction for the QoI to these parameters *and* use this result to choose particular models, discretizations, iterative schemes, etc. over others. Indeed it is possible that the persistence of legacy models that employ tuning parameters in practice is due to their uncertainty to these parameters.

REFERENCES

[1] Taleb, N.N., and Douady, R. (2013). Mathematical definition, mapping, and detection of (anti)fragility. Quantitative Finance, 13(11), 1677?1689.

[2] Taleb, N.N., (2014) *Antifragile: Things that Gain from Disorder*, Random House.