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A Numerical Investigation of Stochastic and Determininstic Upscaling Methods for Permeability Fields; Using SVD and HOSVD Reduction Models

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Due to limitations in computing resources, for simulations with very fine grids it is neither efficient nor possible to obtain exact values of geological properties at all points in the domain (*i.e.*, scale problems) [1, 2, 3]. This is particularly true for large geological domains which span several kilometres. Additionally, it is not possible to obtain reliable and accurate spatial information about geological properties throughout the domain (*i.e.*, uncertainty problems). These are the main drivers for upscaling. Upscaling techniques solve both problems by replacing discrete geological and fluid property values of detailed high resolution domains with coarse descriptions (low resolution) of these properties, while preserving the statistical properties and flow dynamics behaviour [4].

There are several upscaling techniques which fall under different categories [1, 5, 6], however over the last decade, techniques classed as stochastic have received great attention from academic and industrial porous media communities worldwide [7, 8, 9]. One of the reasons for this is because stochastic techniques are robust enough to handle multiphase flow in porous media, and they address scale and uncertainity problems which were previously mentioned.

The aim of this work is to couple stochastic upscaling representations of the permeability field to a novel high-order accurate control volume finite element method (CVFEM) (see [10] for further details of the model formulation) so as to investigate multiphase flow behaviour in highly heterogeneous porous media. The aim is achieved by investigating and comparing the flow dynamics of some stochastic representation models with two other deterministic upscaling techniques (*i.e.*, arithmetic and harmonic mean). Three stochastic representation models are presented in this study. The first is a randomly generated permeability field with Guassian distribution and prescribed probability density function (PDF) which was obtained from a base case (*i.e.*, high-resolution mesh with known permeability distribution). The second and third stochastic representation models are based on singular value decomposition (SVD) and higher order singular value decomposition (HOSVD) [11] model reduction methods.

The base case previously referred to is presented in Figure 1a, and it shows the initial permeability distribution which consists of 4 regions, each region is uniformly randomly generated within a specified range. Initially, the whole domain is fully saturated with a fluid (Fig. 1b), and a wetting phase fluid is driven into the domain from the left-hand side at a constant mass flow rate. Newmann (*i.e.*, no flux) boundary conditions were imposed to the upper and lower borders of the domain. Mixed fluids are recovered from the right-hand face.

Numerical results showed that flow solutions obtained from the stochastic upscaling technique are closer match to the base case compared to the other deterministic upscaling techniques. This is because it retains the heterogeneous nature of the permeability field in each region while the other techniques convert each region from heterogeneous to homogeneous. Additionally, the stochastic upscaling techniques retains second-order statistical properties of the base case while this was lost by the deterministic models.

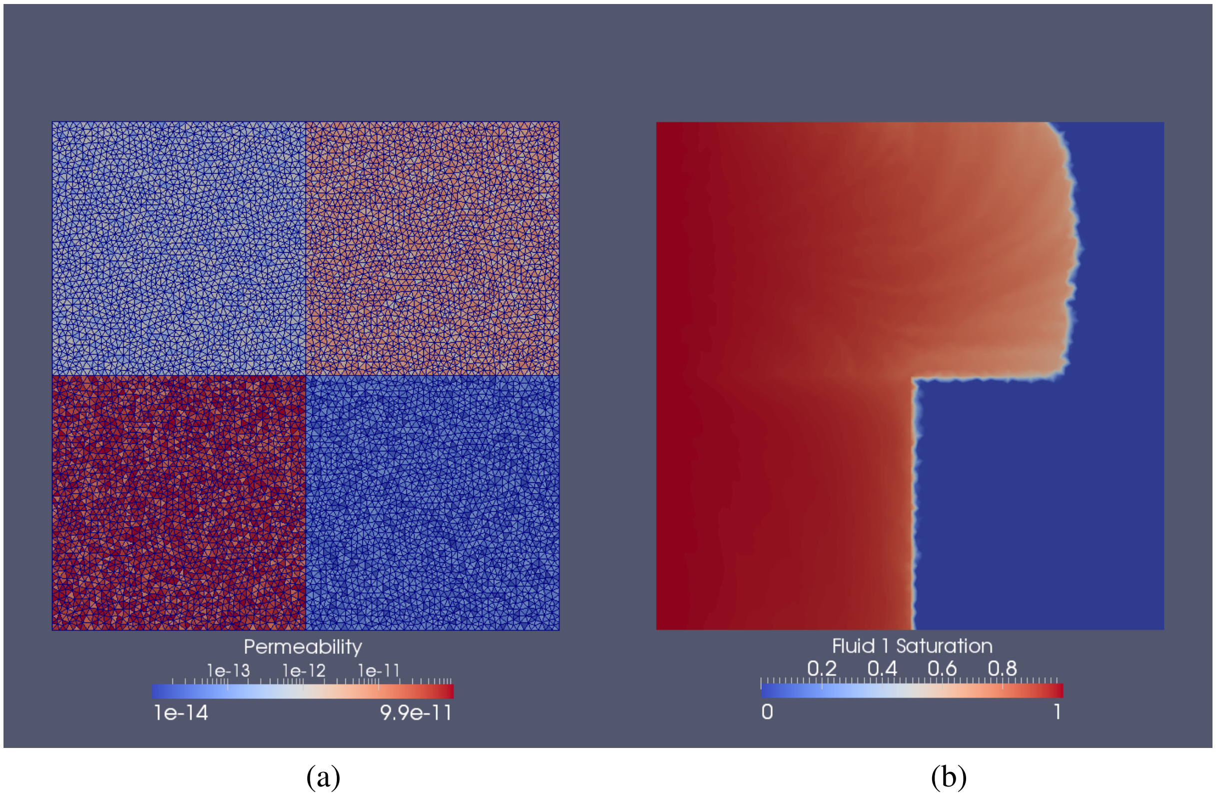

Figure 1: (a) Permeability distribution and (b) phase saturation showing preferential flow pathways.

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