Multi-Grid Preconditioning of Cut-Cell Discretizations

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Cut-Cell methods like Cut-FEM or Unfitted DG have gained a lot of interest in the recent years. In particular for time dependent geometries such methods avoid many practical issues regarding mesh generation and mesh quality and by this reduce the complexity of discretizing complex geometries.

Still this flexibility comes at a price. In particular the construction of efficient preconditioners for cut-cell discretizations is still a challenging problem.

We present recent results on the construction of an efficient multigrid solver for Cut-FEM and Unfitted DG discretizations of elliptic problems. As a first step we have successfully adopted techniques of DG-multi-grid solvers to Unfitted-DG discretizations, by projecting to the conforming first-order subspace and applying algebraic coarsening. The results show optimal complexity with respect to mesh refinement and the overall solver time can compete with that of a fitted discretization.

In this follow-up work we extent these methods further to employ the inherent hierarchy of the structured background mesh used in cut-cell methods. These new techniques are applied to Cut-FEM and Unfitted DG discretizations. We discuss difficulties and possibilities for the construction of geometric multigrid methods for cut-cell discretizations.