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On the Benefit of the Summation-by-parts Property on Interior Nodal Sets

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Originating from the context of finite difference schemes, the summation-by-parts (SBP) property has been used to construct conservative and robust numerical methods for CFD simulations. By the special construction of SBP finite difference schemes, these methods mimic certain properties of the continuous equations in a similar manner as variational schemes such as Galerkin methods. Originally, the nodal sets in the construction of SBP schemes were chosen to contain a sufficient amount of boundary nodes. Recently, the SBP framework has been extended to nodal schemes on exclusively interior nodal sets such as discontinuous Galerkin (DG) schemes on Gauss points in one space dimension or on tensor-product grids, as well as triangular grid DG methods. SBP schemes are successfully applied to split-form equations which are used to establish certain secondary balances, e.g. regarding kinetic energy, either on nodal sets with a sufficient number of boundary nodes or on exclusively interior nodal sets. Although the quadrature rules on interior nodes have a higher degree of exactness, split-form equations require additional boundary terms in order to obtain a consistent and conservative scheme, see [1].

This talk is devoted to a further comparative study of SBP schemes with and without the inclusion of boundary nodes. First, we give some numerical results for the Navier-Stokes equations which demonstrate a possibly higher accuracy of the SBP schemes using Gauss nodes. To gain additional insight, we carry out a Fourier type analysis for various set-ups of the numerical scheme for advection-diffusion equations. Furthermore, the SBP property of DG schemes is used to construct a well-balanced and entropy conservative scheme on the classical Gauss nodes for the shallow water equations.

REFERENCES

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