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ITERATIVE SOLVER FOR SOLVING LARGE-SCALE FRICTIONAL CONTACT PROBLEMS

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In this presentation, we will propose an iterative approach for solving three-dimensional frictional contact problems between elastic bodies, including contact with a rigid body, contact between two or more bodies and also self contact. Since the precise formulation of the elastic part is irrelevant for the description of the algorithm we shall consider a generic case. In practice, however, we will have to deal with a non linear material (for instance a Mooney-Rivlin model). We are interested in solving a finite element approximation of the problem, leading to large-scale non linear discrete problems and, after linearization, to large linear systems and ultimately to calculations needing iterative methods. This also implies that penalty methods, and therefore augmented Lagrangian method, are to be banned because of their negative effect on the condition number of the underlying discrete systems and thus on the convergence of iterative methods. This is in rupture to the mainstream of methods for contact in which augmented Lagrangian is the principal tool. We shall first present the problem and its discretization, this will lead us to describe a general solution algorithm relying on a preconditionner for saddle-point problems which we shall describe in some detail as it is not entirely standard.

The discrete problem employs a Lagrange multiplier for the contact condition. We denote $\underline{\lambda}_n$ and $\underline{\lambda}_T$ its normal and tangential part. (In practice, we use quadratic elements for the displacement and linear elements for the multipliers).

The quasi-static formulation leads to a saddle-point problem, with a Tresca friction condition, coupled with the equation for the threshold $s = \mu\underline{\lambda}_n$ to obtain the Coulomb law. We impose the constraints $\underline{\lambda}_n \geq 0$ and $|\underline{\lambda}_T| \leq s$ at the nodes and we denote the Tresca conditions by $\underline{\lambda} \in \mathcal{T}_s \subseteq \Lambda$. The starting point is thus to solve the problem

$$\inf_{\underline{v}\in V} \sup_{\underline{\lambda}\in\mathcal{T}_s} \mathcal{L}(\underline{v},\underline{\lambda}) = \frac{1}{2}\langle A\underline{v},\ \underline{v}\rangle + I(\lambda_n, \hat{\underline{v}}_n - \hat{\underline{g}}_n) + I(\underline{\lambda}_T, \hat{\underline{v}}_T - \hat{\underline{g}}_T) - \langle \underline{f},\ \underline{v}\rangle, \quad s = \mu\lambda_n \tag{1}$$

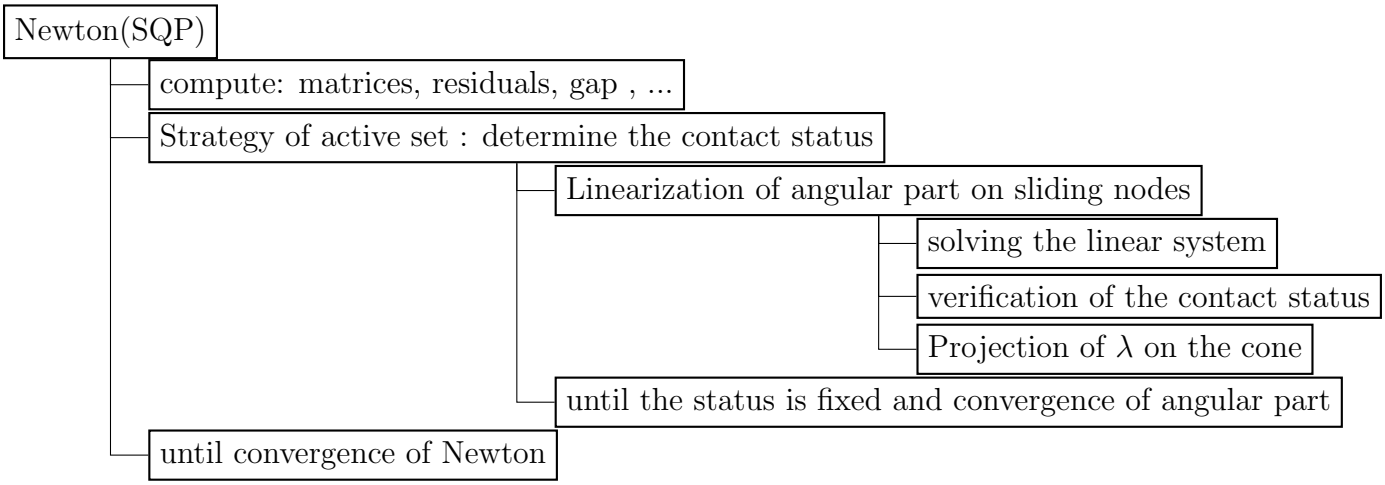
where $I(\cdot,\cdot)$ denotes some numerical integration and where $\hat{v}$ is a projection of $\underline{v}$ on the space of multipliers. Let the matrix B be defined by

$$\langle B\underline{u}, \underline{\lambda}\rangle = \int \underline{u}\,\underline{\lambda}\, ds \tag{2}$$

and M_L be the lumped mass matrix on the space Λ, we then define $\hat{\underline{v}} = M_L^{-1}B\underline{v}$.

The use of a numerical integration, as in [2], is to keep the optimality conditions in $\underline{\lambda}$ pointwise.

The algorithm that we introduce is based on an imbedded loops and can be summarized by the diagram :



We will discuss in this presentation the main line of this algorithm and insist on the solving of the linear system wich is like a saddle-point problem. We will solve it by a GCR method wich is preconditionned by an extension of Uzawa's method. The interest of our technique is that it does not depend on any fixed parameter and the solution of the primal problem can be approximate. Finally, we present an industrial example in biomedical domain which contains up to 15 millions unknows in displacements.

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