

# SOLIDS AND STRUCTURES IN STRAIN GRADIENT ELASTICITY: NUMERICAL COMPARISONS BETWEEN DIMENSION REDUCTION MODELS AND 3D MODELS

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Three-dimensional solid models obeying the strain gradient elasticity theory are compared to the corresponding dimension reduction models – gradient-elastic bar, plane stress/strain, membrane, beam, plate and shell models [1–5] – developed for homogenizing structures composed of (multi-functional) microstructural materials or (hierarchical) architected metamaterials such as lattices. The comparisons, accomplished with respect to the most essential – non-classical – features of the models, serves as a validation for the gradient-elastic dimension reduction models. The comparisons rely on numerical results obtained via isogeometric finite element analysis for both the structural models [1–4] and the three-dimensional solid models. For specific lattice architecture applications (cf. [6]), the strain gradient models are compared to the corresponding (unhomogenized) fine-scale models based on classical elasticity.

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