

# VARIATIONAL UPDATES FOR STRONGLY COUPLED THERMOMECHANICAL PROBLEMS INCLUDING MASS TRANSPORT

Eva M. Andrés<sup>1,2</sup>, Ángel Ortiz-Toranzo<sup>3</sup>, Ignacio Romero<sup>\*1,3</sup>

<sup>1</sup>IMDEA Materials Institute, Getafe, Spain

<sup>2</sup>School of Aerospace Engineering, Technical University of Madrid, Spain

<sup>3</sup>School of Industrial Engineering, Technical University of Madrid, Spain

\*ignacio.romero@upm.es

**Key words:** *Variational updates; coupled problem; thermomechanics*

Variational update formulas have proven an effective method for developing robust integration algorithms in nonlinear solid mechanics of dissipative media. Many references describe such methods for constitutive models such as elastoplasticity, viscoelasticity, damage, etc. See, among others, [1, 2].

The extension of variational updates to thermomechanical problems is much more involved and only after the pioneering work of Yang *et al.* [3] these kind of methods could be extended to coupled systems. In this key reference, and in order to recover a variational statement of the equations, it was necessary to introduce an artificial time re-scaling. Later methods also inherit this scaling in their formulations [4].

In the current work we describe a novel variational update that simplifies and extends the formulation of Yang *et al.*. First, it simplifies the existing formulation showing that an alternative choice of the thermodynamic forces makes the time re-scaling unnecessary. Also, it extends previous formulations because we are able to include mass transport in the same framework, making the three-field rate problem completely variational. Numerical simulations will be shown to illustrate the performance of the proposed method.

## REFERENCES

- [1] M. Ortiz and L. Stainier. The variational formulation of viscoplastic constitutive updates. *Computer Methods in Applied Mechanics and Engineering*, 171(3):419–444, 1999.
- [2] C. Miehe, J. Schotte, and M. Lambrecht. Homogenization of inelastic solid materials at finite strains based on incremental minimization principles. Application to the texture analysis of polycrystals. *Journal of the Mechanics and Physics of Solids*, 50(10):2123–2167, 2002.
- [3] Q. Yang, L. Stainier, and M. Ortiz. A variational formulation of the coupled thermo-mechanical boundary-value problem for general dissipative solids. *Journal of the Mechanics and Physics of Solids*, 54:401–424, 2006.
- [4] A. Bartels, T. Bartel, M. Canadija, and J. Mosler. On the thermomechanical coupling in dissipative materials: A variational approach for generalized standard materials. *Journal of the Mechanics and Physics of Solids*, 82:218–234, 2015.