6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

Subcell *a posteriori* limitation of high-order DG scheme through flux reconstruction correction

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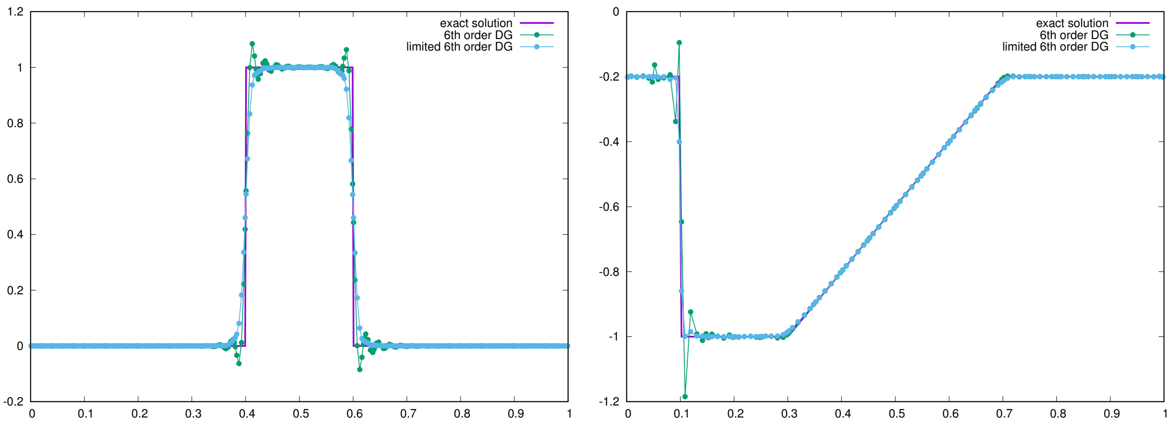
Keywords: *A posteriori limitation, subcell limitation, arbitrary high-order discontinuous Galerkin scheme, Flux reconstruction (CPR), hyperbolic conservation laws*

The discontinuous Galerkin (DG) method, introduced by Reed and Hill and further developed in a well-known series of paper of Cockburn and Shu, have become more and more popular these past decades. This is mainly due to its high accuracy while keeping the stencil compact, along with other good properties as L_2 stability and *hp*-adaptivity. However, dealing with discontinuities, DG method needs some sort of nonlinear limiting to avoid spurious oscillations due to the Gibbs phenomenon. Furthermore, numerical approximations may generate non-admissible solution (negative density or pressure in the case of gas dynamics for instance), which may lead to nonlinear instability or crash of the code. These fundamental issues has been extensively tackled in the past, and there is thus a vast literature on limiters. Recently, some new techniques, referred to as *a posteriori* limitation, have arisen [1, 2]. Our method falls into this category.

The main idea motivating this work is to preserve as much as possible the high accuracy and the very precise subcell resolution of the DG scheme. Consequently, an *a posteriori* correction will only be applied locally at the subcell scale where it is needed. Do to so, through the use of flux reconstruction [4], also refereed to as correction procedure via reconstruction (CPR) [5], we first prove that it is possible to rewrite DG scheme as a subcell finite volume scheme. This analytical part provides us with a so-called DG reconstructed flux. Then, at each time step, we compute a DG candidate solution and check if this solution is admissible (for instance positive, non-oscillating, entropic, . . .). If it is the case, we go further in time. Otherwise, we return to the previous time step and correct locally, at the subcell scale, this solution. This is why it is refereed to as *a posteriori* limitation. In [2, 3], if a solution in a cell is detected as bad, the cell is subdivided into subcells and a robust first-order finite volume is performed on each subcell. Then, through these subcell mean values, a high-order polynomial is reconstructed on the primal cell. Here, we only modify locally at the subcell level the solution where it is needed. Practically, if the solution on a subcell has been detected as bad, we substitute the DG reconstructed flux on its boundary by a robust first-order numerical flux. And for subcell detected as admissible, we keep the high-order reconstructed flux which allow us to retain the very high accurate resolution of the DG scheme at the subcell resolution.

Numerical results on various type problems and test cases will be presented to assess the good performance of the design limiting algorithm, see **Figure 1**.



(a) Linear advection on 50 cells after 10 periods

(b) Burgers equation on 20 cells

Figure 1: Limited and unlimited 6th order DG scheme : subcell mean values.

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