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# STRAIN INVARIANCE IN GEOMETRICALLY EXACT 3D BEAMS: RELATIONSHIP BETWEEN APPROACHES ON TWO DIFFERENT CONFIGURATION MANIFOLDS

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The kinematics of large motion of 3D beams is untypical in that its configuration manifold is not a 3D vector space, but rather a product of such a space and a three-parameter Lie group of proper orthogonal transformations SO(3). This observation has led to beam finite elements for modelling large 3D motion [1] in which interpolation of the rotation parameters turns out to be an issue deserving particular attention if the approximated strain fields are required to inherit the important physical property of strain-invariance under rigid-body motion [2]. While considerable body of research has been naturally directed towards avoiding these complexities by simply avoiding 3D rotations, a complementary idea has been presented in [3], whereby the whole configuration space is defined as a non-linear manifold. In this way all the field variables may be processed in a unique manner and It has been shown in [3] that this leads to an explicit law governing the evolution of the 6D configuration tensors owing to the fact that this manifold may be defined not only as another Lie group, but in fact as such a Lie group in which a closed-form exponentional relationship with its algebra exists. While such an approach is pretty elegant when it comes to actual implementation, it necessarily inherits, and in fact exacerbates, the above undesirable side effect. Unless properly handled, the ensuing formulation is likely to suffer from severe lack of strain-invariance, showing up even in 2D problems. A solution is provided for a somewhat different representation of the configuration tensor in [4] leading to highly efficient finite elements. In this contribution, we will expose a full parallelism between the evolutions of proper orthogonal tensors and configuration tensors which extends all the way to providing a strain-invariant configuration-tensor interpolation, completely analogous to that applicable to higher-order beam elements [2]. Special attention will be paid to the exponential maps and their derivatives and inverse derivatives in both approaches in order to assess the configuration-tensor method not only in terms of accuracy, but also in terms of its ability to handle large load increments within the iterative Newton-Raphson solution procedure.

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