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Reduced-order model of the BGK equation based on Proper Orthogonal Decomposition and Optimal Transportation

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A reduced-order model of the BGK equation leading to fast and accurate computations is presented. The BGK model describes the dynamics of a gas flow in both hydrodynamic and rarefied regimes:

$$\frac{\partial f}{\partial t}(\mathbf{x}, \xi, t) + \xi \cdot \nabla_{\mathbf{x}} f(\mathbf{x}, \xi, t) = \frac{M_f(\mathbf{x}, \xi, t) - f(\mathbf{x}, \xi, t)}{\tau(\mathbf{x}, t)} \tag{1}$$

where f is the density distribution function representing the density of gas particles at point $\mathbf{x} \in \mathbb{R}^3$, velocity $\xi \in \mathbb{R}^3$ and time $t \in \mathbb{R}$, and M_f is the Maxwellian distribution function. In the reduced-order model, the distribution functions are approximated in velocity space by a small number of basis functions Φ_n computed offline:

$$\widetilde{f}(\mathbf{x}, \xi, t) = \sum_{n=1}^{N} a_n^f(\mathbf{x}, t)\Phi_n(\xi) \qquad \text{and} \qquad \widetilde{M}_f(\mathbf{x}, \xi, t) = \sum_{n=1}^{N} a_n^M(\mathbf{x}, t)\Phi_n(\xi) \tag{2}$$

In the offline phase, the BGK equation is sampled in order to collect information on the distribution functions. Then, optimal transportation provides additional samples by interpolating the distribution functions to complete the sampling. Finally, the basis functions are built by Proper Orthogonal Decomposition. During the online phase, the offline knowledge is used to compute approximations of the density distribution function at low cost. The BGK equation is projected onto the basis functions, leading to an hyperbolic system of partial differential equations. Moreover, the projection is modified to conserve mass, momentum and energy of the gas. The resulting system is then solved by an IMEX Runge-Kutta scheme in time and a finite-volume scheme in space to compute the coefficients a_n^f and a_n^M. The results show the significant reduction of the computational cost and the accuracy of the reduced-order model:

Figure 1: Vertical plate in a flow at Kn = 0.0345. Streamlines and horizontal velocity component (left panel). Prediction error (right panel). The database is created for a Mach number of 0.63 and the predicted solutions are for Mach 0.23,0.33,0.43,0.53,0.63. The number of modes is either 15 or 20. Computational time is divided by 45.

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