Model Order Reduction for Hyperbolic Conservation Laws

R. Crisovan[*], R. Abgrall[1]

[*] UZH I-MATH, Winterthurerstrasse 190, CH-8057 Zrich, roxana.crisovan@math.uzh.ch,
`https://www.math.uzh.ch/index.php?assistenten&key1=10625`
[1] UZH I-MATH, Winterthurerstrasse 190, CH-8057 Zrich, remi.abgrall@math.uzh.ch,
`www.math.uzh.ch/index.php?id=1819&key1=8882`

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Reduced basis (RB) methods can alleviate the cost of repeated simulations with limited computational resources and are directly based on the underlying high-dimensional model that results from standard finite element, finite volume or finite difference formulation. These methods restrict the solution to be contained in a subspace of the underlying high-dimensional space, this subspace being determined by an optimal reduced basis in a training phase. Thus, a large number of degrees of freedom (say millions) are represented by only a few number of coefficients in the representation of the full solution in terms of the reduced basis vectors, leading to important computational savings.

The case of systems of hyperbolic conservation laws is a special, challenging one in comparison with the elliptic and parabolic ones, for which most of the RB theory was developed and numerically applied. In this context, moving waves and discontinuities such as shocks will depend on different parameter settings and they will evolve in time. Hence, accurate surrogates have to be developed, in order to be able to capture the evolution of the discontinuous solutions, which implicitly involve non-linearities.

We proposed new model order reduction techniques, and in conjunction with the well known theory of RB methods to demonstrate that they will fit better when reducing problems with sharp gradient and shocks. Firstly, we proved that model order reduction (MOR) using L^1-norm minimization of the residual leads to a more accurate reduced solution, L^1-norm being a natural norm for evolution problems which involves discontinuities. In order to reduce the Kolmogorov N-width of the solution manifold, we consider a dictionary approach based on no compression when generating the reduced basis functions. In order to emphasize the accuracy of the method we are also providing robust error estimators for the scalar problems in the case of monotone schemes and we illustrate the behaviour of L^1-norm minimization based on a dictionary approach on linear and nonlinear problems, both in one and two dimensional cases.

Secondly, the parameter dependency of the system of hyperbolic conservation laws means that for different parameter inputs, the position and the shape of the shock is changing. Because of this behaviour, we might encounter discrepancies in the reduced solution. In order to fix this oscillatory behaviour, we are making use of calibration ideas applying them in this context of RB methods with focus on the steady two dimensional Euler equation around an airfoil.

Last part of our work focuses on MOR methods for parametric nonlinear hyperbolic conservation laws with applications in uncertainty quantification (UQ). To generate a RB space, we want to find a low dimensional good approximation of the high fidelity functional space. For this, we are using methods as PODEI-Greedy algorithm, by extending the empirical interpolation method (EIM) basis functions and the POD-Greedy basis functions in a synchronized way.