6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

Error Controlled Numerical Model Reduction in FE^2 Analysis of Transient Heat Flow

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Keywords: *model reduction, multiscale modeling, error estimation*

One standard approach to multiscale modeling is the so-called FE^2 procedure, where the classic constitutive relation is replaced by a boundary value problem on a Representative Volume Element (RVE) comprising the underlying microscale features. It is well realized that straight-forward use of the FE^2-strategy can be computationally intractable for a fine macroscale mesh. Therefore, it is of interest to reduce the cost of solving the individual RVE-problem(s) by introducing some kind of reduced basis, here denoted Numerical Model Reduction (NMR). However, it is important to note that the richness of the reduced basis will determine the accuracy of the solution, which calls for error control.

In this presentation, we consider a two-scale formulation of linear transient heat conduction as a model problem. We follow Aggestam et al. [1] and use Spectral Decomposition to obtain a reduced basis, replacing RVE-problem with a set of uncoupled ordinary differential equations in time.

For the error estimation, we focus solely on the error due to the reduced basis and ignore time- and space-discretization errors. We aim for guaranteed, explicit bounds on the error in (i) energy norm and (ii) an arbitrary quantity of interest (QoI) within the realm of goal-oriented error estimation. As a "workhorse" for the error computation, we thereby introduce an associated (non-physical) symmetrized variational problem in space-time. We derive a low cost estimator, which, in particular, requires no extra modes than the ones used for the reduced basis approximation.

Numerical investigations concerns FE^2-problems, in 1D and 2D on the macroscale. The microscale is always modeled in full 3D. The results show that we obtain a good estimate of the error, both in terms of energy norm and in terms of different QoIs, with a cost that is negligible compared to the reduced basis approximation.

REFERENCES

[1] E. Aggestam, F. Larsson, K. Runesson, and F. Ekre. Numerical model reduction with error control in computational homogenization of transient heat flow. *Computer Methods in Applied Mechanics and Engineering*, 326:193–222, 2017.