

## Higher Order Methods using Algebraic Dual Spaces

V. Jain<sup>1</sup>, Y. Zhang<sup>1</sup>, A. Palha<sup>2</sup> and M. Gerritsma<sup>1</sup>

<sup>1</sup> Faculty of Aerospace Engineering, TU Delft, Delft, The Netherlands.  
email : {V.Jain, Y.Zhang-14, M.I.Gerritsma}@tudelft.nl

<sup>2</sup> Department of Mechanical Engineering, TU Eindhoven, Eindhoven, The Netherlands.  
email : A.Palha@tue.nl

**Keywords:** *Dual isogeometric basis, Finite element method, Mimetic method*

In the framework of FEM, we present a discretization method with two main novel ideas. First, we define degrees of freedom on all types of geometric objects, (in case of  $\mathbb{R}^2$ ) i.e. nodes, edges and surfaces. This ensures that certain parts of the algebraic discretization are topologically exact [1-3]. Second, we make explicit use of algebraic dual spaces. This allows for very sparse higher order methods which are independent of the size and shape of the grid [4]. In particular, we look at the application of these methods in constraint minimization problems and mixed formulations such as - mixed formulation of the Poisson problem, mixed formulations in structural mechanics, and incompressible flow problems.

In this work, we propose dual isogeometric basis functions that make use of algebraic dual spaces for splines. We present their application on i) the mixed formulation of the Poisson problem; and ii) the mixed formulation for the plane stress linear elasticity problem in structural mechanics. In both applications we see that the constraints are satisfied exactly up to machine precision. Furthermore, we observe optimal convergences of physical variables and a very sparse matrix system. The condition numbers for matrix system are much less, and also their growth is much lower, as compared to conventional methods.

### REFERENCES

- [1] J. Kreeft, and M. Gerritsma, *Mixed mimetic spectral element method for Stokes flow: A pointwise divergence-free solution*. Journal of Computational Physics, 240 (2013) 284-309.
- [2] A. Palha, P. P. Rebelo, R. Hiemstra, J. Kreeft, and M. Gerritsma, *Physics-compatible discretization techniques on single and dual grids, with application to the Poisson equation of volume forms*. Journal of Computational Physics, 257 (2014) 1394-1422.
- [3] P. Bochev, and M. Gerritsma, *A spectral mimetic least-squares method*, CAMWA, 68 (2014) 1480-1502.
- [4] V. Jain, Y. Zhang, A. Palha and M. Gerritsma, *Construction and application of algebraic dual polynomial representations for finite element methods*, arXiv:1712.09472, (2017).