

Parallel Multigrid Solver for Finite Elements using B-Splines

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Magnetic confinement fusion aims to harvest energy from the fusion of small atomic nuclei, typically Deuterium and Tritium, which are heated to the plasma state and confined by a toroidal magnetic field configuration. The Tokamak concept is at present the best developed magnetic confinement device, in which the confining magnetic field is produced by external magnetic field coils and a toroidal plasma current.

In order to support the exascale plan for Fusion energy, a parallel, robust and optimal geometric Multigrid is needed as a building block for Kinetic/Gyrokinetic models and scalable MagnetoHydrodynamics solvers using Physics Based Preconditioning methods.

High order Finite Elements methods, specially on (block) structured grids, are a good compromise to meet both high resolution (spectral convergence) and optimal scalability on the massively parallel hardware architecture of modern and future supercomputers. However, it has been shown that matrices arising from High Order BSplines discretizations present some pathologies in high frequencies. Such pathologies cannot be treated by the classical Multigrid method, which only deals with low frequencies.

Following a previous work of Donatelli *et al* [1], to overcome these limitations, we study a solution yielding an optimal Multigrid method for B-Splines by studying the associated symbol using the Generalized Locally Toeplitz (GLT)[2]. A post-smoother based on GLT is then used after every V-cycle of the multigrid algorithm.

In this talk, we will describe a parallel implementation of a geometric GLT based Multigrid in the context of Isogeometric analysis. Scalability will be discussed and studied for MPI and a hybrid MPI-OpenMP parallelization.

REFERENCES

- [1] Marco Donatelli, Carlo Garoni, Carla Manni, Stefano Serra-Capizzano, and Hendrik Speleers. Robust and optimal multi-iterative techniques for iga galerkin linear systems. *Computer Methods in Applied Mechanics and Engineering*, 284:230 – 264, 2015. Isogeometric Analysis Special Issue.
- [2] Stefano Serra-Capizzano. The glt class as a generalized fourier analysis and applications. *Linear Algebra and its Applications*, 419(1):180 – 233, 2006.