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From low to high-order Hybridizable Discontinuous Galerkin for Computational Vademecums

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Hybridizable Discontinuous Galerkin [1–5] has in recent years demonstrated its ability to solve a wide range of engineering problems. The HDG method is able to provide the optimal approximation properties that are characteristic of mixed methods, including the possibility to build a superconvergent solution (and thus, error estimation), whilst retaining the advantages of DG methods. In addition, HDG methods are known to reduce the globally coupled degrees of freedom, when compared to other DG methods. Moreover, HDG is competitive compared to the traditional continuous Galerkin (CG) method and has comparable costs (in terms of floating point operations) to CG [6]. Other advantages of HDG, such as block structured information and element-by-element operations must be exploited to improve its performance compared to CG because parallelism and memory access are crucial for the final runtime.

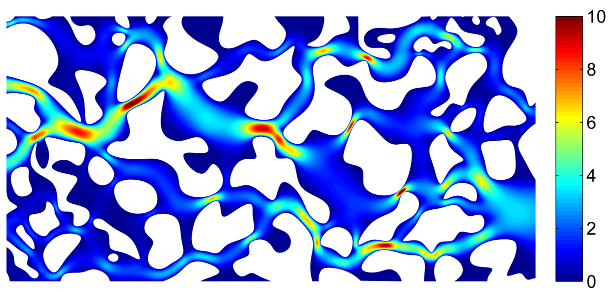

Figure 1: Velocity magnitude of flow in porous media.

When high-order approximations are competitive, HDG can be combined with NURBS-enhanced finite element method (NEFEM) [7] to design a superior and reliable strategy where degree adaption allows to attain the desired accuracy, see Figure 1. Moreover, in the context of computational mechanics, an alternative formulation to impose physical natural boundary conditions and strongly the symmetry of the strain tensor is proposed by means of the well-known Voigt notation for symmetric tensors. Optimal convergence of the mixed variable is retrieved for low-order elements and a novel local post-process procedure leading to a superconvergent velocity field is advanced.

Other scenarios are however more suited for low-order approximations. Under this perspective, a new finite volume paradigm is presented [8]. It is called face-centered finite

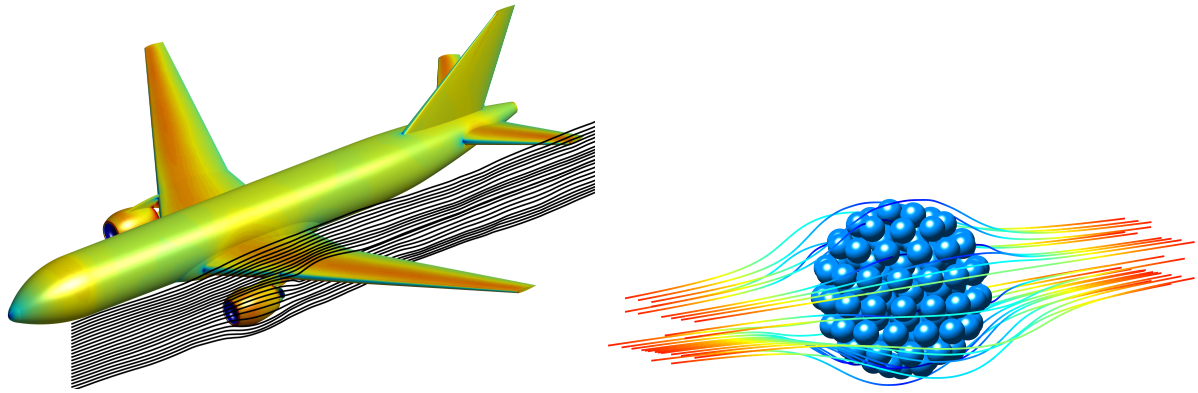

Figure 2: Irrotational flow past full aircraft (left). Viscous streamlines around spherical particles (right).

volume (FCFV) and is based on an HDG method with constant degree of approximation. First order convergence on both the solution and its gradient is obtained without a reconstruction of the gradients. Therefore, contrary to other finite volume methodologies, the accuracy of the FCFV method is not compromised in the presence of highly stretched or distorted elements, see Figure fg:FCFV.

Finally, the advantages of HDG in a model order reduction context are also exploited. The combination of HDG with the proper generalized decomposition (PGD) provides an a priori computational vademecum which allows an accurate and fast evaluation of elliptic problems with parameterized geometries.

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