6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

Explicit Runge-Kutta methods for index-2 and index-3 differential algebraic equations arising in incompressible flow

B. Sanderse [1], A.E.P. Veldman [2]

[1] Centrum Wiskunde & Informatica (CWI), Amsterdam, The Netherlands
[2] Bernoulli Institute for Mathematics, University of Groningen, The Netherlands

Keywords: *Differential-algebraic equations, Runge-Kutta method, Incompressible flow*

Many computational physics problems can be modelled by partial differential equations (PDEs) with constraints. In particular, we are interested in single-phase and multi-phase incompressible fluid flow problems, in which the constraint is that the velocity field is divergence-free. After discretizing the PDEs in space, a differential-algebraic equation (DAE) system is obtained. In previous work we have analyzed the accuracy of explicit Runge-Kutta methods for the single-phase incompressible Navier-Stokes equations, which form an index-2 DAE [2]. In the current work we consider the extension to multi-phase incompressible flow problems in pipelines and channels, where a slightly different constraint (sum of volume fractions equals 1) leads to a DAE with index 3. Existing time integration methods for this system lack either conservation, accuracy, or constraint-consistency.

We propose a new time integration method that is consistent with the constraints of the index-3 DAE system (including two hidden constraints), by basing it on 'half-explicit' Runge-Kutta methods [1]: explicit for the mass and momentum equations and implicit for the pressure. The resulting method is (i) *constraint-consistent*: exact conservation of the volume constraint and the incompressibility constraint; (ii) *accurate*: high order temporal accuracy for differential and algebraic variables; (iii) *conservative*: the original mass and momentum equations are solved, so that the proper shock conditions are satisfied; (iv) *efficient*: the only implicit part is the pressure Poisson equation, and the time step for the explicit part is restricted by a benign CFL condition based on the convective wave speeds.

Several testcases show the effectiveness of the time-integration methods: Kelvin-Helmholtz instabilities in a pipeline, liquid sloshing in a tank, and ramp-up of gas production in a multi-phase pipeline.

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