

Numerical Homogenization for Linear Elasticity in Translation Invariant Spaces

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In this talk we introduce a generalization of the truncated Fourier series approach of Moulinec and Suquet [1] and the discretization using constant finite elements by Brisard and Dormieux [2].

The method we present generalizes these discretization methods to anisotropic spaces of translates, i.e. spaces generated by translating a single function on an anisotropic lattice on the torus [3]. Such ansatz functions include periodized Box splines (and thus simplified finite elements) and de la Vallée Poussin means. The truncated Fourier series approach emerges as a special case concerning the equality of the discrete solutions of the variational formulation of the PDE and the Lippmann-Schwinger equation. We further present a convergence theorem.

The choice of the anisotropic lattice in generating the translation invariant space allows to adapt the sampling grid to dominant directions occurring in the composite material. (Anisotropic) de la Vallée Poussin means provide a way to smoothen these directions independent from each other, which can further reduce disturbances from jumps at interface boundaries, as will be demonstrated on numerical examples.

REFERENCES

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