ASPECTS OF CONSTANT TRACTION BOUNDARY CONDITIONS IN COMPUTATIONAL HOMOGENIZATION

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The objective of this presentation is to elaborate on various aspects of constant traction boundary conditions within a computational homogenization framework at finite deformation. In the past decades computational homogenization has proven to be a powerful strategy to compute the overall response of continua. Central to computational homogenization is the Hill–Mandel condition. The Hill–Mandel condition is fulfilled via imposing displacement boundary conditions (DBC), periodic boundary conditions (PBC) or traction boundary conditions (TBC) collectively referred to as canonical boundary conditions. While DBC and PBC are widely implemented, TBC remains poorly understood, with a few exceptions. The main issue with TBC is the singularity of the stiffness matrix due to rigid body motions. The primary feature of this presentation is to propose a generic strategy to implement TBC in the context of computational homogenization at finite strains. To eliminate rigid body motions, we introduce the concept of *semi-Dirichlet boundary conditions* [1]. Semi-Dirichlet boundary conditions are non-homogeneous Dirichlet-type constraints that simultaneously satisfy the Neumann-type conditions. A key feature of the proposed methodology is its applicability for both strain-driven as well as stress-driven homogenization. The performance of the proposed scheme is demonstrated via a series of numerical examples. The constant traction boundary conditions are particularly interesting since

- TBC can be rigorously formulated in accordance with minimization of averaged incremental energy [2],
- Somewhat incorrectly, TBC is sometimes referred to as being ill-posed,
- Numerical simulations show that the performance of TBC is superior to both DBC and PBC,
- TBC shows less sensitivity with respect to the microstructure of the material.

In summary, this presentation sheds light on the micro-to-macro transition for both stress-driven as well as strain-driven homogenization frameworks using TBC and employing semi-Dirichlet constraints instead of Lagrange multipliers. The similarities, differences, pros and cons of both approaches are discussed, as well.

References

[1] A. Javili, S. Saeb, P. Steinmann (2017). Aspects of implementing constant traction boundary conditions in computational homogenization via semi-Dirichlet boundary conditions. Computational Mechanics 59:21–35

[2] C. Miehe (2003). Computational micro-to-macro transitions for discretized micro-structures of heterogeneous materials at finite strains based on the minimization of averaged incremental energy. Computer Methods in Applied Mechanics and Engineering 192: 559–591.