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SIMULATING INTERACTIONS OF SPATIAL NONLINEAR WAVES IN A SHALLOW LAYER OF VISCOUS LIQUID

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Propagation of plane nonlinear surface waves in shallow water is traditionally described by the well-known KdV equation. However, there is no single model equation for the case of three-dimensional finite-amplitude perturbations. Their dynamics can be described only by systems of several equations that have to take into account both the disturbances of the free surface and these of mean velocity vector of the liquid. The known systems involve this vector even in the linear terms of all equations. Here we provide a model, which is more convenient for the analysis of the spatial nonlinear waves interactions.

We assume that the bottom is gently sloping and the emerging boundary layers remain thin. Hence, the flow induced by the wave is potential everywhere except for the thin boundary layer. By analogy with the approach used in [1] for an inviscid liquid we reduce the hydrodynamic equations to the basic evolution equation of the wave type. The specific feature of the equation is that the average values of horizontal components of the liquid velocity are included only in one term of the second order of smallness. Therefore, we may use a simple linear approximation of the continuity equation to determine this velocity.

The system is solved numerically with the use of Fourier method. After discretization the auxiliary linear continuity equation becomes a simple algebraic expression for Fourier-amplitudes of velocity potential at a fixed time moment. Viscous effects are taken into account through an integral term in the main wave-type equation and are calculated by trapezoidal method. The resulting system of ODEs for Fourier amplitudes of free surface perturbations is solved by the Runge–Kutta method of the fourth order of accuracy.

A significant number of problems of nonlinear disturbance transformations are calculated by means of the new approach as well as with the use of the so called two-dimensional Boussinesq equation, derived under the assumption that nonlinear perturbations travel mainly in one direction. The disrepancies between results obtained by the two mentioned models are shown to be comparable with the contribution of nonlinearity.

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REFERENCES

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