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The Logarithmic Finite Element method

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The Logarithmic finite element (LogFE) method extends the Ritz-Galerkin method to approximations on a non-linear finite-dimensional manifold in the infinite-dimensional solution space. Formulating the interpolant on the logarithmic space allows for a novel treatment of the rotational component of the deformation, making this approach especially suitable for geometrically exact formulations involving large rotations.

Using homogeneous coordinates, the logarithms of transformation matrices representing rotations and translations constitute basis vectors of a linear subspace composed of the logarithmic representations of affine transformations. The admissible transformations, resulting from linear combinations of logarithmic shape functions, generate a Lie algebra.

Given an appropriate formulation of the finite elements, local degrees of freedom can be linked to global degrees of freedom and boundary conditions, and the interpolant is given by an immersion of the space of degrees of freedom into the configuration space. Thus, the LogFE method satisfies the criteria for FE models as given by Ciarlet [1].

The basic characteristics of the LogFE method will be illustrated by examining the model of a two-dimensional geometrically exact planar Bernoulli beam element [2]. Extensions of the model to Timoshenko kinematics and the three-dimensional case will be discussed, as well as a co-rotational extension of the LogFE beam formulation. The co-rotational formulation enables the model to exactly represent pure rigid body motions and ensures that spurious high-order deformation components vanish with mesh refinement, thus satisfying the interpolation theorem for finite elements [3].

REFERENCES

[1] Ciarlet, P.G. *The Finite Element Method for Elliptic Problems.* 1st edition, 2nd printing, North-Holland, Amsterdam, 1979.

[2] Schröppel, C. and Wackerfuß, J. “Introducing the Logarithmic finite element method: a geometrically exact planar Bernoulli beam element.” *Advanced Modeling and Simulation in Engineering Sciences*, 3 (1), 2016.

[3] Oden, J.T. and Reddy, J.N. *An Introduction to the Mathematical Theory of Finite Elements.* Corrected republication, Dover, Mineola, 2011 (1976).